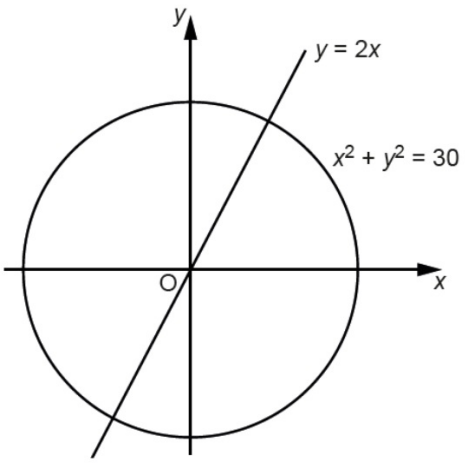
A57 (H) Simultaneous Equations - Circles and Straight Lines

OCR

17 Find the exact coordinates of the two intersections of the line $y = 2x$ and the circle $x^2 + y^2 = 30$.

A57

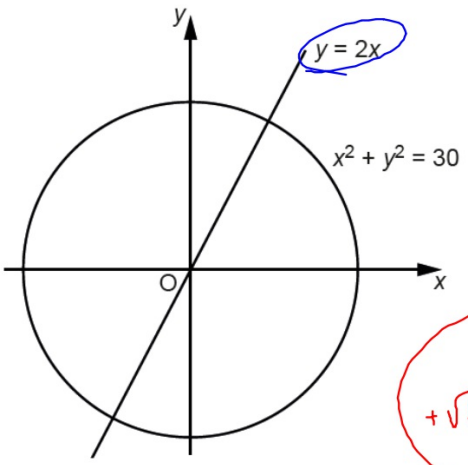


Not to scale

(..... , ...) and (..... , ...) **[5]**

17 Find the exact coordinates of the two intersections of the line $y = 2x$ and the circle $x^2 + y^2 = 30$.

A57



$x^2 + (2x)^2 = 30$

$x^2 + 4x^2 = 30$

$5x^2 = 30$

$x^2 = 6$

$x = \pm\sqrt{6}$

$y = 2x$

$+\sqrt{6}$ $y = 2 \times \sqrt{6}$

$y = 2\sqrt{6}$

$y = 2x$

$x = -\sqrt{6}$

$y = 2 \times -\sqrt{6}$

$= -2\sqrt{6}$

($\sqrt{6}$, $2\sqrt{6}$) and ($-\sqrt{6}$, $-2\sqrt{6}$) **[5]** ✓

EDEXCEL

20 Solve algebraically the simultaneous equations

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$$x^2 + y^2 = 25$$
$$y - 2x = 5$$

(Total for Question 20 is 5 marks)

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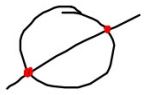
20 Solve algebraically the simultaneous equations

$x^2 + y^2 = 25$ ... circle $x^2+y^2=r^2$ ... $r=5$

$y - 2x = 5$ ... straight line

$y = 2x + 5$



$x^2 + (2x+5)^2 = 25$

$x^2 + 4x^2 + 20x + 25 = 25$

$5x^2 + 20x + 25 - 25 = 0$

$5x^2 + 20x = 0$

$(2x+5)(2x+5)$

$4x^2 + 10x + 10x + 25$

$4x^2 + 20x + 25$

$5x^2 + 20x = 0$

$a = 5$

$b = 20$

$c = 0$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{-20 \pm \sqrt{400}}{10}$

$\frac{-20+20}{10}$ and $\frac{-20-20}{10}$

$x = 0$ and $x = -4$

ans

$x = 0, y = 5$

$x = -4, y = -3$

$y - 2x = 5$

$y - 0 = 5$

$y = 5$

$y - 2x = 5$

$y - -8 = 5$

$-3 + 8 = 5$

(Total for Question 20 is 5 marks)

20 Solve algebraically

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$$x^2 + y^2 = 18$$
$$x - 2y = -3$$

.....

(Total for Question 20 is 5 marks)

20 Solve algebraically

$x^2 + y^2 = 18$

$x - 2y = -3$

$2 \times 3/5 = 6/5 \cdots 1\frac{1}{5} \cdots 1.2$ ✓

$x^2 + y^2 = 18$

$x - 2y = -3$

$x = 2y - 3$

$x - 2y = -3$

$\square - 1.2 = -3$

$\square + 1.2 = -3$

-4.2

$(2y-3)^2 + y^2 = 18$ … quadratic

$(2y-3)(2y-3)$

$4y^2 - 6y - 6y + 9$

$4y^2 - 12y + 9$

$4y^2 - 12y + 9 + y^2 = 18$

$ay^2 + by + c = 0$

$5y^2 - 12y + 9 - 18 = 0$

$5y^2 - 12y - 9 = 0$

-9

$+3 \quad -3$

$+9 \quad -1$

$+3y - 15y = -12y$ ✓

$(5y+3)(y-3) = 0$

$5y + 3 = 0$

$5y = -3$

$y = -3/5$

$x = -4.2$ ✓

or $y - 3 = 0$

$y = 3$

$x = 3$ ✓

✓

(Total for Question 20 is 5 marks)

20 Solve algebraically the simultaneous equations

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$$x^2 + y^2 = 25$$
$$y - 3x = 13$$

.....

(Total for Question 20 is 5 marks)

20 Solve algebraically the simultaneous equations

$x^2 + y^2 = 25$

$y - 3x = 13$

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$y = 3x + 13$

$x^2 + y^2 = 25$

$x^2 + 9x^2 + 78x + 169 = 25$

$10x^2 + 78x + 169 - 25 = 0$

$10x^2 + 78x + 144 = 0$

$5x^2 + 39x + 72 = 0$

72 → 24, 3

$(5x + 24)(x + 3) = 0$

$5x + 24 = 0$ | $x + 3 = 0$

$5x = -24$ | $x = -3$

$x = -\frac{24}{5}$

$x = -4.8$

$4.8 \times 3 = 14.4$

$(3x + 13)^2$

$(3x + 13)(3x + 13)$

$9x^2 + 39x + 39x + 169$

$9x^2 + 78x + 169$

$x = -4.8$

$y = 3(-4.8) + 13$

$y = -14.4 + 13$

$y = -1.4$

$x = -4.8$ and $y = -1.4$

$x = -3$

$y = 3(-3) + 13$

$y = -9 + 13$

$y = 4$

and $x = -3$ and $y = 4$ ✓

(Total for Question 20 is 5 marks)

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19 Solve algebraically the simultaneous equations

A57

$$2x^2 - y^2 = 17$$
$$x + 2y = 1$$

(Total for Question 19 is 5 marks)

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19 Solve algebraically the simultaneous equations

$$2x^2 - y^2 = 17$$
$$x + 2y = 1 \quad \ldots \; x = 1 - 2y$$

A57

$2(1-2y)^2 - y^2 = 17$

$2(4y^2 - 4y + 1) - y^2 = 17$

$8y^2 - 8y + 2 - y^2 - 17 = 0$

$7y^2 - 8y - 15 = 0$

$a = 7$, $b = -8$, $C = -15$

$y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \dfrac{8 \pm \sqrt{64 - -420}}{14}$

$y = \dfrac{8 \pm \sqrt{484}}{14}$

$(1-2y)(1-2y)$

$1 - 2y - 2y + 4y^2$

$= 4y^2 - 4y + 1$

$y = \frac{15}{7}$	$y = -1$
$x + 2y = 1$	$x + 2y = 1$
$x + 2\left(\frac{15}{7}\right) = 1$	$x + 2(-1) = 1$
$x + \frac{30}{7} = 1$	$x - 2 = 1$
$x = -\frac{23}{7}$	$x = 3$

Ans
$x = 3, \; y = -1$
$x = -\frac{23}{7}, \; y = \frac{15}{7}$

(Total for Question 19 is 5 marks)

AQA

27 The line $y = 3x + p$ and the circle $x^2 + y^2 = 53$ intersect at points A and B.
p is a positive integer.

27 (a) Show that the x-coordinates of points A and B satisfy the equation

$$10x^2 + 6px + p^2 - 53 = 0$$

A57

[3 marks]

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27 (b) The coordinates of A are (2, 7)

A57

Work out the coordinates of B.

You **must** show your working.

[5 marks]

Answer (_____ , _____)

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27 The line $y = 3x + p$ and the circle $x^2 + y^2 = 53$ intersect at points A and B.
p is a positive integer.

27 (a) Show that the x-coordinates of points A and B satisfy the equation

$$10x^2 + 6px + p^2 - 53 = 0$$

A57

[3 marks]

$x^2 + (3x+p)^2 = 53$

$x^2 + 9x^2 + 6xp + p^2 = 53$

$10x^2 + 6xp + p^2 - 53 = 0$

$(3x+p)(3x+p)$

$9x^2 + 3xp + 3xp + p^2$

$9x^2 + 6xp + p^2$

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27 (b) The coordinates of *A* are (2, 7) x y

A57

Work out the coordinates of *B*.

You **must** show your working.

[5 marks]

$y = 3x + 1$

$x^2 + y^2 = 53$

$y = 3x + p$

$7 = 6 + p$

$p = 1$

$(3x+1)(3x+1)$

$9x^2 + 3x + 3x + 1$

$9x^2 + 6x + 1$

$4ac$

$4 \times 5 \times -26$

$= -520$

$a = 5$

$b = 3$

$c = -26$

$x^2 + (3x+1)^2 = 53$

$x^2 + 9x^2 + 6x + 1 - 53 = 0$

$10x^2 + 6x - 52 = 0$

$5x^2 + 3x - 26 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-3 \pm \sqrt{9 - -520}}{10}$

$x = \frac{-3 \pm \sqrt{529}}{10}$

$x = \frac{-3 \pm \sqrt{529}}{10}$

$x = -2.6$

$y = -6.8$

$y = 3x + 1$

$y = 3(-2.6) + 1$

Answer (−2.6 , −6.8)