

A48-49 (H) Completing the Square

OCR

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16 Write $x^2 - 10x + 16$ in the form $(x + a)^2 + b$.

A48

..... [3]

16 Write $x^2 - 10x + 16$ in the form $(x + a)^2 + b$.

A48

$$(x - 5)^2 - 9$$

$$-5^2 = \underline{\underline{+25}} \quad \xrightarrow{-9} \quad +16$$

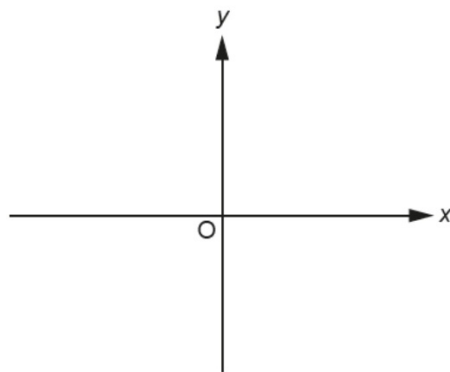
$$\underline{\underline{(x - 5)^2 - 9}}$$

[3]

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- 19 (a) Sketch the graph of $y = (x - 2)^2 - 3$.
Show the coordinates of any turning points.

A49



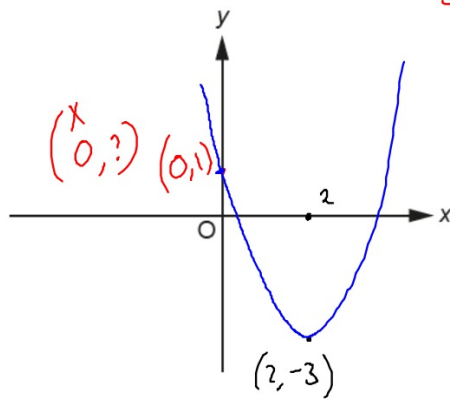
[3]

$$x=2 \quad y=-3$$

- 19 (a) Sketch the graph of $y = (x-2)^2 - 3$.
Show the coordinates of any turning points.

A49

turning point



extra

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$$y = (x-2)^2 - 3$$

$$y = (x-2)(x-2) - 3$$

$$y = x^2 - 4x + 4 - 3$$

$$y = x^2 - 4x + \underline{1}$$

$$y = 1$$

[3]

16 (a) Write $x^2 - 6x + 20$ in the form $(x - a)^2 + b$.

A48

(a) [3]

(b) Write down the turning point of the graph of $y = x^2 - 6x + 20$.

A49

(b) (..... ,) [2]

16 (a) Write $x^2 - 6x + 20$ in the form $(x - a)^2 + b$.

A48

$$-3x-3 \quad (x-3)^2 + 11$$
$$-3^2 = 9$$

$$(x-3)(x-3)$$
$$x^2 - 6x + 9 + 11$$

(a) $(x-3)^2 + 11$ [3]

(b) Write down the turning point of the graph of $y = x^2 - 6x + 20$.

A49

$$U_{\min} \quad x=3$$
$$y=11$$

(b) (3 , 11) [2]

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15 (a) Write $x^2 - 8x + 25$ in the form $(x - a)^2 + b$.

(a) **[3]**

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(b) Write down the coordinates of the turning point of the graph of $y = x^2 - 8x + 25$.

(b) (..... ,) [2]

(c) Hence describe the single transformation which maps the graph of $y = x^2$ onto the graph of $y = x^2 - 8x + 25$.

15 (a) Write $x^2 - 8x + 25$ in the form $(x - a)^2 + b$.

$$(x - 4)^2 + 9 \checkmark$$

$$-4^2 = +16$$

$$\begin{array}{cc} 16 & 25 \\ \underbrace{\hspace{1.5cm}} & \\ +9 & \end{array}$$

(a) $(x - 4)^2 + 9$ [3]

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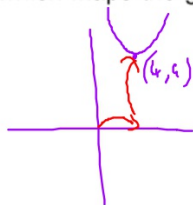
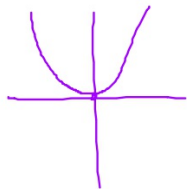
(b) Write down the coordinates of the turning point of the graph of $y = x^2 - 8x + 25$.

$(x-4)^2 + 9$
zero
Min point $(4, 9)$



(b) (.....4.....,9.....) [2]

(c) Hence describe the single transformation which maps the graph of $y = x^2$ onto the graph of $y = x^2 - 8x + 25$.



Translate $\begin{pmatrix} 4 \\ 9 \end{pmatrix}$ ✓

18 (a) (i) Write $x^2 + 4x - 16$ in the form $(x + a)^2 - b$.

A48

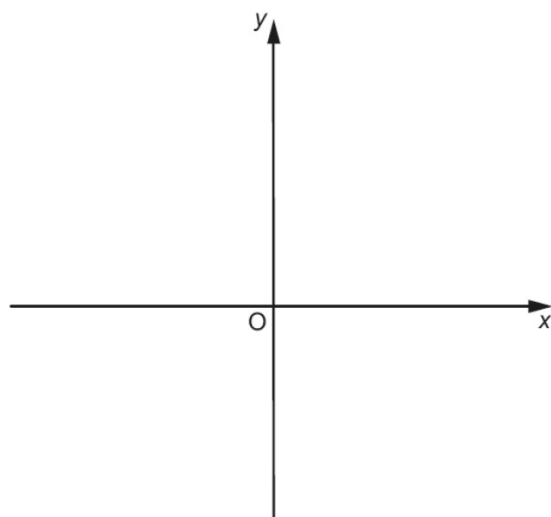
(a)(i) [3]

(ii) Solve the equation $x^2 + 4x - 16 = 0$.
Give your answers in surd form as simply as possible.

(ii) $x = \dots\dots\dots$ or $x = \dots\dots\dots$ [4]

(b) Sketch the graph of $y = x^2 + 4x - 16$, showing clearly the coordinates of any turning points.

A49



[3]

- 18 (a) (i) Write $x^2 + 4x - 16$ in the form $(x + a)^2 - b$.

A48

$$2^2 = 4$$

$$(x + 2)^2 - 20$$

(a)(i) $(x + 2)^2 - 20$ [3]

- (ii) Solve the equation $x^2 + 4x - 16 = 0$.
Give your answers in surd form as simply as possible.

$$\begin{array}{l} \sqrt{20} \\ \sqrt{4}\sqrt{5} \\ 2\sqrt{5} \end{array}$$

$$(x + 2)^2 - 20 = 0$$

$$(x + 2)^2 = 20$$

$$x + 2 = \pm\sqrt{20}$$

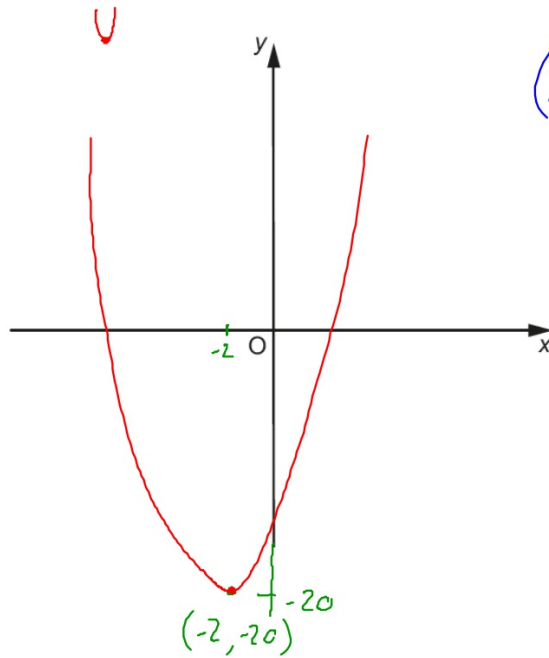
$$x = \pm\sqrt{20} - 2$$

$$\rightarrow \pm 2\sqrt{5} - 2$$

(ii) $x = 2\sqrt{5} - 2$ or $x = -2\sqrt{5} - 2$ [4]

(b) Sketch the graph of $y = x^2 + 4x - 16$, showing clearly the coordinates of any turning points.

A49



$$(x+2)^2 - 20$$

min

$x \rightarrow (-2)$ $y \rightarrow (-20)$



[3]

Edexcel

11 Write $x^2 + 2x - 8$ in the form $(x + m)^2 + n$
where m and n are integers.

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.....
(Total for Question 11 is 2 marks)

11 Write $x^2 + 2x - 8$ in the form $(x + m)^2 + n$ where m and n are integers.

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↳ Completing the square.

$$x^2 + 2x - 8$$
$$(x + 1)^2 - 9 \checkmark$$

$$\begin{matrix} 2 \\ 1 = 1 \end{matrix}$$

$$(x+1)^2 - 9$$

(Total for Question 11 is 2 marks)

$$(x+1)(x+1)$$
$$x^2 + 1x + 1x + 1$$
$$x^2 + 2x + 1$$

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17 Solve $x^2 - 6x - 8 = 0$

Write your answer in the form $a \pm \sqrt{b}$ where a and b are integers.

.....
(Total for Question 17 is 3 marks)

17 Solve $x^2 - 6x - 8 = 0$... complete the square.

Write your answer in the form $a \pm \sqrt{b}$ where a and b are integers.

$$x^2 - 6x - 8 = 0$$

$$(x-3)^2 - 17 = 0$$

$$(x-3)(x-3)$$

$$x^2 - 3x - 3x + 9$$

$$x^2 - 6x + 9$$

Solve

$$(x-3)^2 - 17 = 0$$

$$(x-3)^2 = 17$$

$$x-3 = \pm\sqrt{17}$$

$$x = 3 \pm \sqrt{17}$$

(Total for Question 17 is 3 marks)

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23 (a) Write $2x^2 + 16x + 35$ in the form $a(x + b)^2 + c$ where a , b , and c are integers.

.....
(3)

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(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 2x^2 + 16x + 35$

.....
(1)

(Total for Question 23 is 4 marks)

23 (a) Write $2x^2 + 16x + 35$ in the form $a(x + b)^2 + c$ where a , b , and c are integers.

$$2[x^2 + 8x + 17.5]$$

$$2[(x+4)^2 + 1.5]$$

$$2(x+4)^2 + 3$$

Complete the square
 x^2

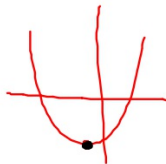
$$(x+4)^2$$

$$(x+4)(x+4)$$

$$x^2 + 4x + 4x + 16$$

$$x^2 + 8x + 16$$

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 2x^2 + 16x + 35$

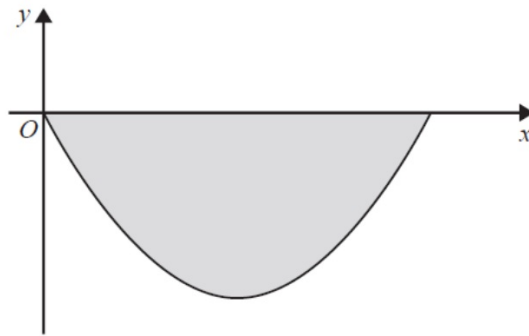


x	y
(-4	, 3)

✓

15 Here is a sketch of a vertical cross section through the centre of a bowl.

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The cross section is the shaded region between the curve and the x -axis.

The curve has equation $y = \frac{x^2}{10} - 3x$ where x and y are both measured in centimetres.

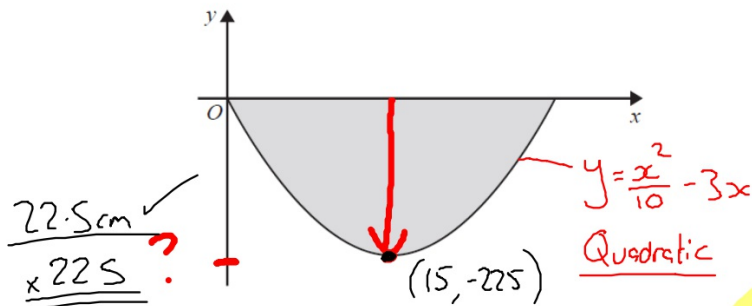
Find the depth of the bowl.

..... cm

(Total for Question 15 is 4 marks)

15 Here is a sketch of a vertical cross section through the centre of a bowl.

Video created by W Neill



The cross section is the shaded region between the curve and the x -axis.

The curve has equation $y = \frac{x^2}{10} - 3x$ where x and y are both measured in centimetres.

Find the depth of the bowl.

$$\begin{aligned} (50 - 15)^2 \\ -15^2 = 225 \end{aligned}$$

$$\begin{aligned} y &= \frac{x^2}{10} - 3x \\ 10y &= x^2 - 30x \\ x^2 - 30x + 0 &= 0 \end{aligned}$$

$$(x - 15)^2 - 225$$

Small as possible

$$10y = 225 \text{ cm}$$

$$y = 22.5 \text{ cm}$$

(Total for Question 15 is 4 marks)

Video created by W Neill

- 19** By completing the square, find the coordinates of the turning point of the curve with equation $y = x^2 + 10x + 18$
You must show all your working.

(.....,))

(Total for Question 19 is 3 marks)

- 19 By completing the square, find the coordinates of the turning point of the curve with equation $y = x^2 + 10x + 18$. You must show all your working.

$$x^2 + 10x + 18$$

$$(x + 5)^2 - 7$$

$$(x+5)(x+5)$$

$$x^2 + 5x + 5x + 25$$

$$x^2 + 10x + 25$$

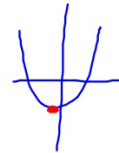
$$\text{from } 25 \xrightarrow{-7} +18$$

gms

$$(x+5)^2 - 7$$

needs to be zero

$$x = -5$$



$$(-5, -7)$$

(Total for Question 19 is 3 marks)

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18 $(x - 8)(x + 4) = (x - a)^2 + b$ for all values of x .

Find the value of a and the value of b .

$a = \dots\dots\dots$

$b = \dots\dots\dots$

(Total for Question 18 is 3 marks)

18 $(x - 8)(x + 4) = (x - a)^2 + b$ for all values of x .

Find the value of a and the value of b .

$$\begin{aligned} &(x-2)(x-2)^2 \\ &\underline{x^2 - 4x + 4} \end{aligned}$$

$$\begin{aligned} &(x-8)(x+4) \\ &x^2 + 4x - 8x - 32 \\ &x^2 - 4x - 32 \end{aligned} \quad \rightarrow \quad \begin{aligned} &\underline{x^2 - 4x - 32} \\ &(x-2)^2 - 36 \\ &(x-2)^2 - 36 \end{aligned}$$

$$a = \underline{2}$$

$$b = \underline{-36}$$

(Total for Question 18 is 3 marks)

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16 Solve $(x - 2)^2 = 3$

Give your solutions correct to 3 significant figures.

(Total for Question 16 is 2 marks)

16 Solve $(x-2)^2 = 3$

Give your solutions correct to 3 significant figures.

$$(x-2)^2 = 3$$

$$x-2 = \pm\sqrt{3}$$

$$x = \pm\sqrt{3} + 2$$

$$3.73 \checkmark$$

$$0.268 \checkmark$$

(Total for Question 16 is 2 marks)

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13 Write $x^2 + 6x - 7$ in the form $(x + a)^2 + b$ where a and b are integers.

.....
(Total for Question 13 is 2 marks)

13 Write $x^2 + 6x - 7$ in the form $(x + a)^2 + b$ where a and b are integers.

$$(x+3)^2 - 16$$

$$+3 = +9$$

$$(x+3)^2 - 16 \checkmark$$

(Total for Question 13 is 2 marks)

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(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 2x^2 + 16x + 35$

.....
(1)

(Total for Question 23 is 4 marks)

17 Here is a sketch of a curve.

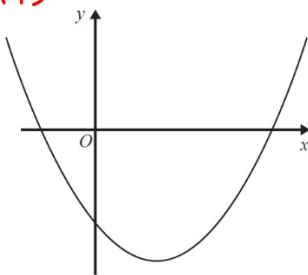
A48

A49

The equation of the curve is $y = x^2 + ax + b$ where a and b are integers.

The points $(0, -5)$ and $(5, 0)$ lie on the curve.

Find the coordinates of the turning point of the curve.



(.....,))

(Total for Question 17 is 4 marks)

17 Here is a sketch of a curve.

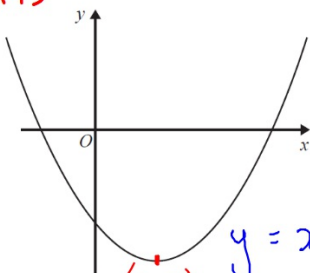
The equation of the curve is $y = x^2 + ax + b$ where a and b are integers.

The points $(0, -5)$ and $(5, 0)$ lie on the curve.

Find the coordinates of the turning point of the curve.

A48

A49



$$y = x^2 - 4x - 5$$

$$x^2 - 4x - 5$$

$$(x-2)^2 - 9$$

$$\begin{matrix} \text{min} \\ x = 2 \end{matrix}$$

$$y = -9$$

$$-2^2 = +4 \rightarrow -5$$

$$\begin{matrix} x & y \\ (0, & -5) \end{matrix} \dots y = x^2 + ax + b$$

$$-5 = 0^2 + 0 + b \dots b = -5$$

$$\begin{matrix} x & y \\ (5, & 0) \end{matrix} \dots 0 = x^2 + ax - 5$$

$$0 = 25 + 5a - 5$$

$$0 = 20 + 5a$$

$$-20 = 5a$$

$$\frac{-20}{5} = a$$

$$-4 = a$$

$$(\dots 2, \dots -9 \dots)$$

(Total for Question 17 is 4 marks)

AQA

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27 Prove that $x^2 + x + 1$ is always positive.

[3 marks]

A48

27 Prove that $x^2 + |x| + 1$ is always positive.

[3 marks]

A48

$$\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\left(x + \frac{1}{2}\right)\left(x + \frac{1}{2}\right)$$

$$x^2 + \frac{1}{2}x + \frac{1}{2}x + \frac{1}{4}$$

$$x^2 + |x| + \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

positive²
as anything²
is positive

positive

positive + positive ✓
= positive

23 The equation of a curve is $y = (x + 3)^2 + 5$

A48 Circle the coordinates of the turning point.

A49

[1 mark]

(5, 3)

(5, -3)

(3, 5)

(-3, 5)

23

The equation of a curve is $y = (x + 3)^2 + 5$

A48

Circle the coordinates of the turning point.

A49

[1 mark]

(5, 3)

(5, -3)

(3, 5)

(-3, 5)

$$\begin{array}{l} \textcircled{0} \\ (x + 3)^2 \\ \underline{x = -3} \end{array} \quad \begin{array}{l} y \\ + 5 \\ \hline \end{array}$$

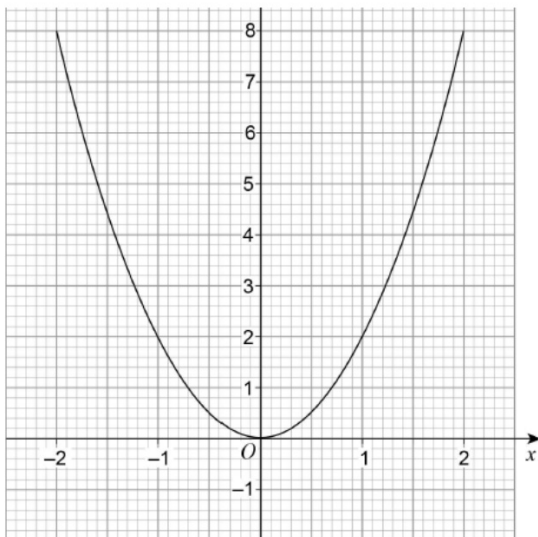
(-3, 5)

21 (a) Meera is using a **graphical** method to solve $2x^2 - 3x = 0$

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She draws the graph of $y = 2x^2$ and a straight line graph on the same grid.

Here is the graph of $y = 2x^2$



Complete her method to solve $2x^2 - 3x = 0$ [2 marks]

Answer _____

21 (b) Levi is solving $2x^2 + 5x = 0$

He uses this method.

A46

A48

$$2x^2 + 5x = 0 \quad \text{subtract } 5x \text{ from both sides}$$

$$2x^2 = -5x \quad \text{divide both sides by } x$$

$$2x = -5 \quad \text{divide both sides by 2}$$

$$x = -2.5$$

Evaluate his method and his answer.

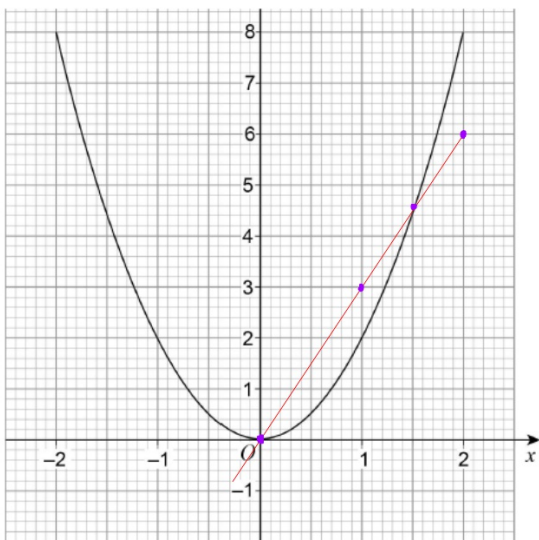
[2 marks]

21 (a) Meera is using a graphical method to solve $2x^2 - 3x = 0$

Video created by W Neill

She draws the graph of $y = 2x^2$ and a straight line graph on the same grid.

Here is the graph of $y = 2x^2$ ✓



Complete her method to solve $2x^2 - 3x = 0$ [2 marks]

$$2x^2 - 3x = 0$$

$$2x^2 = 3x$$

$$y = \uparrow \quad y = \uparrow$$

Draw $y = 3x + 0$

$$y = mx + c$$



Answer $x = 0$, $x = 1.5$ ✓

Video created by W Neill

21 (b) Levi is solving $2x^2 + 5x = 0$

Quadratic

He uses this method.

A46

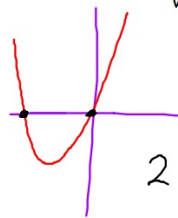
A48

$$2x^2 + 5x = 0 \quad \text{subtract } 5x \text{ from both sides}$$

$$2x^2 = -5x \quad \text{divide both sides by } x$$

$$2x = -5 \quad \text{divide both sides by 2}$$

$$x = -2.5$$



2 solutions

Evaluate his method and his answer.

$$2x^2 + 5x = 0$$

[2 marks]

$$x(2x + 5) = 0$$

missed

$$\frac{x = 0 \checkmark$$

$$2x + 5 = 0$$

$$2x = -5$$

$$x = -5/2 \checkmark$$

27 (b) Without expanding any brackets,

A48

show how to work out the **exact** solutions of $9(x + 3)^2 = 4$
Give the solutions.

[3 marks]

27 (b) Without expanding any brackets,

A48

show how to work out the **exact** solutions of $9(x + 3)^2 = 4$
Give the solutions.

[3 marks]

$$9(x+3)^2 = 4$$

$$(x+3)^2 = \frac{4}{9}$$

$$x+3 = \sqrt{\frac{4}{9}}$$

$$x+3 = \frac{2}{3}$$

$$x = \frac{2}{3} - 3$$

$$x = -2\frac{1}{3} \checkmark$$

$$\sqrt{\frac{4}{9}} = \frac{2}{3}$$

20

Curve P has equation $y = 2(x - 1)^2 - 5$

A49

Curve Q is a reflection in the y -axis of curve P.

A30

Work out the equation of curve Q.

Give your answer in the form $y = ax^2 + bx + c$ where a , b and c are integers.

[3 marks]

Answer _____

20

Curve P has equation $y = 2(x - 1)^2 - 5$

A49

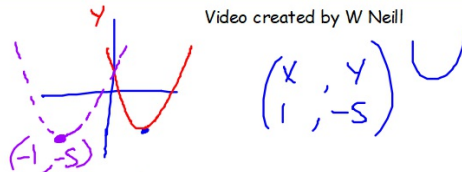
Curve Q is a reflection in the y -axis of curve P.

A30

Work out the equation of curve Q.

Give your answer in the form $y = ax^2 + bx + c$ where a , b and c are integers.

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[3 marks]

$$y = 2(-x - 1)^2 - 5$$

$$y = 2(x^2 + 2x + 1) - 5$$

$$y = 2x^2 + 4x + 2 - 5$$

$$y = 2x^2 + 4x - 3$$

$$(-x - 1)(-x - 1)$$

	-x	-1	
-x	x ²	+1x	
-1	+1x	+1	
			x ² + 2x + 1

Answer $y = 2x^2 + 4x - 3$ ✓

