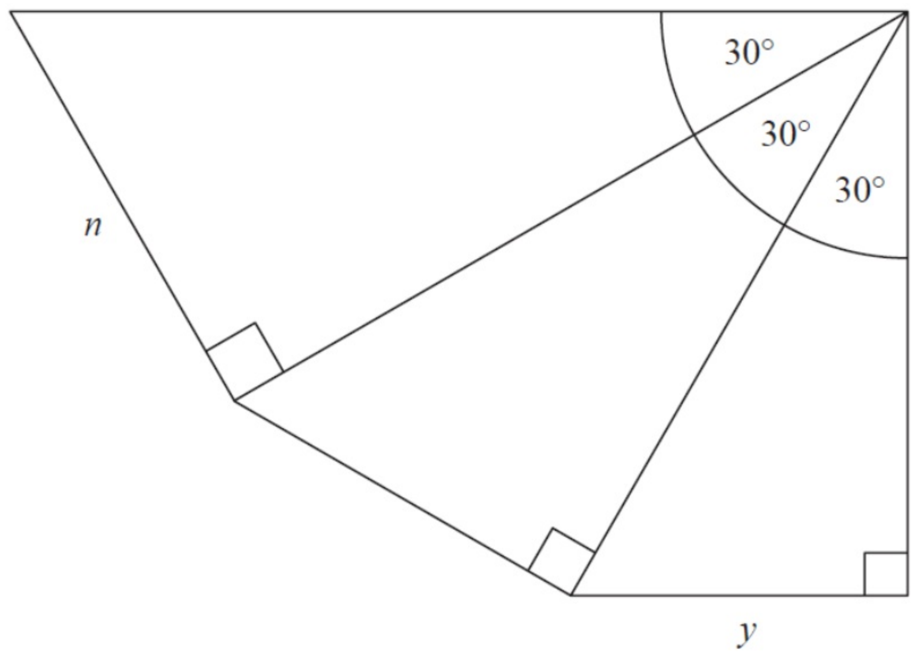
G66...Difficult Algebraic Proof

OCR

Edexcel

Created by W Neill

20



The diagram shows three right-angled triangles.

Prove that $y = \frac{3}{4}n$

(Total for Question 20 is 4 marks)

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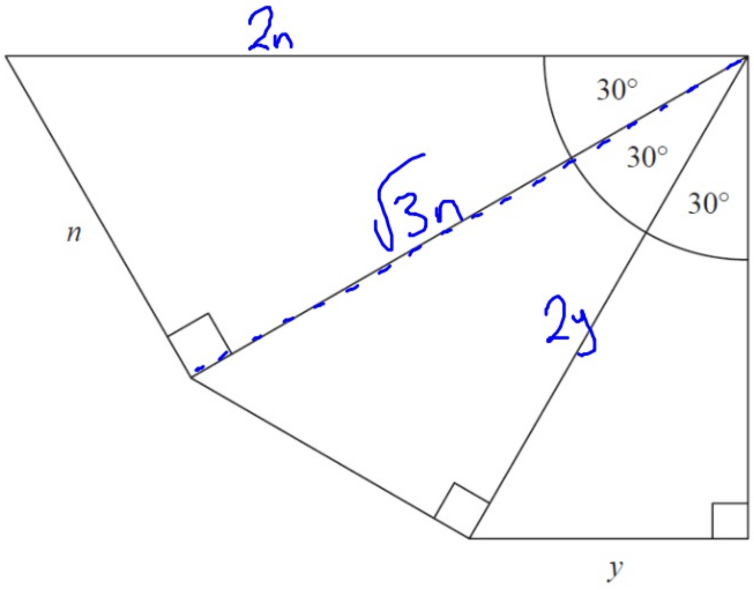
20



The diagram shows three right-angled triangles.

Prove that $y = \frac{3}{4}n$

$\sin 30° = \frac{1}{2}$ $\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$

SOH CAH TOA

$= H = \frac{y}{\frac{1}{2}}$

$H = 2y$

$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}n}{1} = 2y$

$\frac{3n}{2} = 2y$

$\frac{3n}{4} = y$

(Total for Question 20 is 4 marks)

21 The diagram shows an acute-angled triangle ABC.

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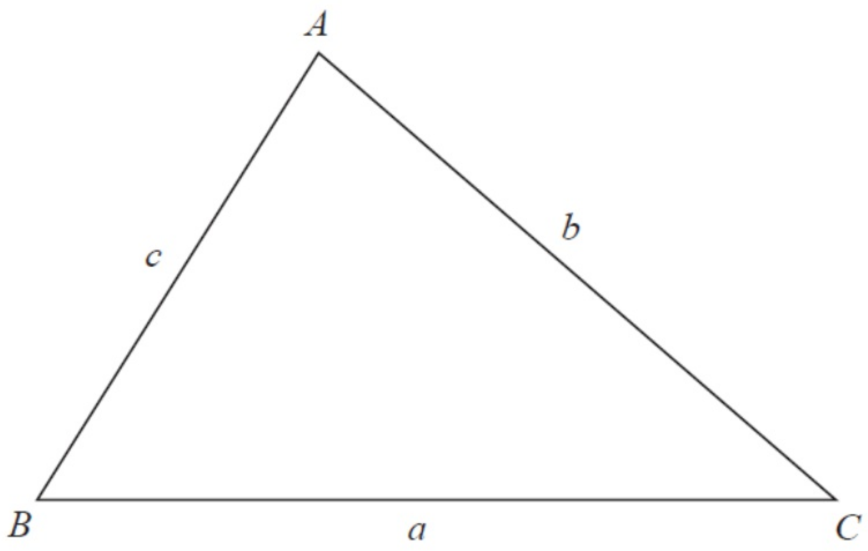

Prove that area of triangle $ABC = \frac{1}{2}ab\sin C$

(Total for Question 21 is 3 marks)

21 The diagram shows an acute-angled triangle *ABC*.

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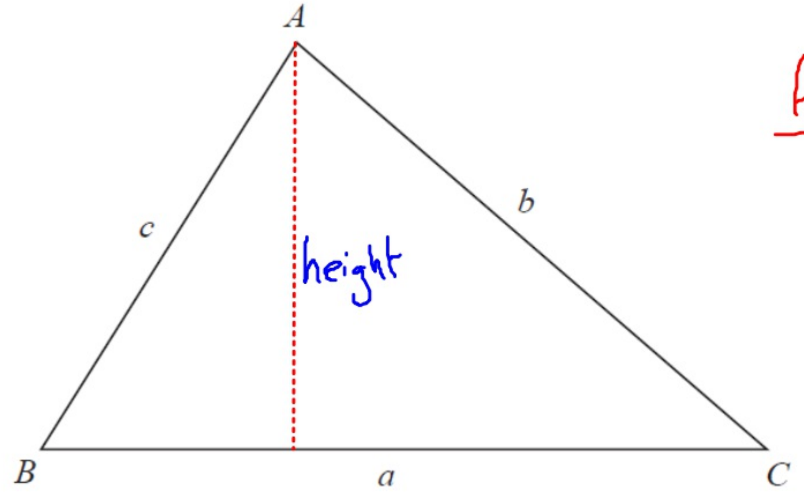


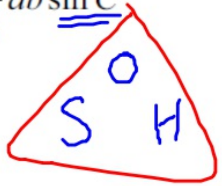
Area of triangle

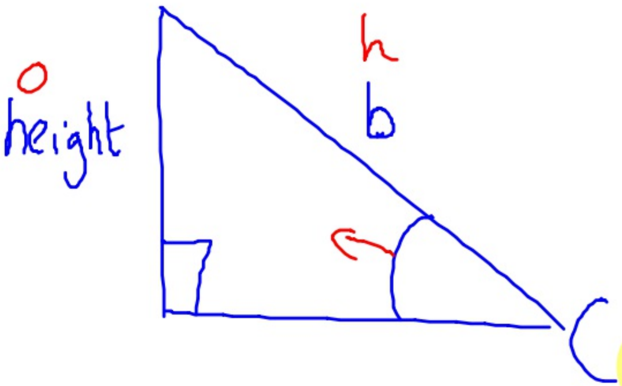
½ x base x height

½ x a x height

½ x a x bSinC

Prove that area of triangle $ABC = \frac{1}{2}ab\sin C$





$\text{Sin}C = \frac{\text{height}}{b}$

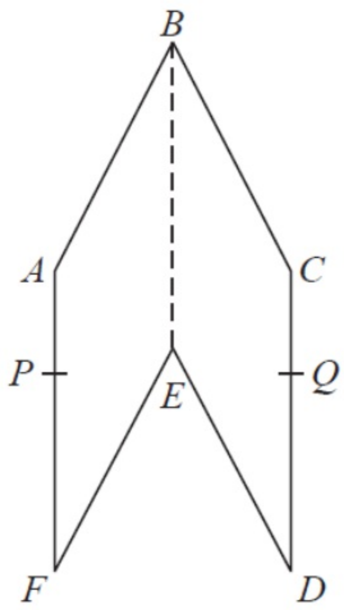
bSinC = height

½ abSinC

area of a triangle ✓

(Total for Question 21 is 3 marks)

22 The diagram shows a hexagon *ABCDEF*.



Video created by W Neill

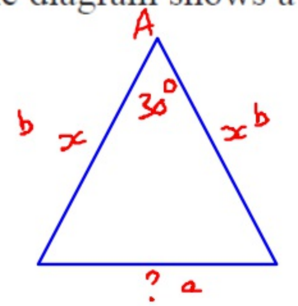
ABEF and *CBED* are congruent parallelograms where $AB = BC = x$ cm.
P is the point on *AF* and *Q* is the point on *CD* such that $BP = BQ = 10$ cm.

Given that angle $ABC = 30°$,

prove that $\cos PBQ = 1 - \dfrac{(2-\sqrt{3})}{200}x^2$

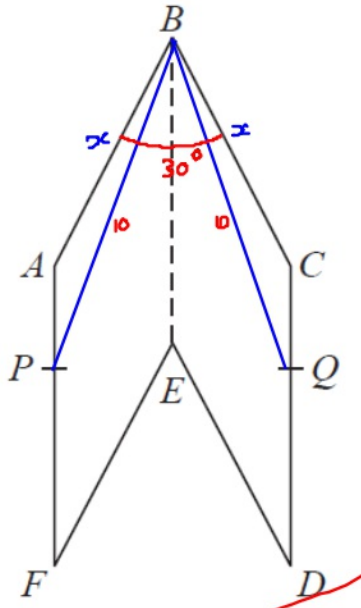
22 The diagram shows a hexagon *ABCDEF*.

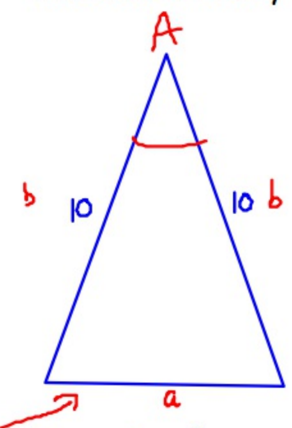
Video created by W Neill



$\cos A = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$





$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = x^2 + x^2 - 2x^2\left(\frac{\sqrt{3}}{2}\right)$

$a^2 = 2x^2 - x^2(\sqrt{3})$

$a^2 = 2x^2 - \sqrt{3}x^2$

$a^2 = x^2(2 - \sqrt{3})$

ABEF and *CBED* are congruent parallelograms where *AB* = *BC* = *x* cm.
P is the point on *AF* and *Q* is the point on *CD* such that *BP* = *BQ* = 10 cm.

Given that angle *ABC* = 30°,

prove that $\cos PBQ = 1 - \frac{(2-\sqrt{3})}{200}x^2$

$a^2 = b^2 + c^2 - 2bc\cos A$

$\frac{a^2 - b^2 - c^2}{-2bc} = \cos A$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \frac{100 + 100 - x^2(2 - \sqrt{3})}{200}$

$\cos A = \frac{200 - (2 - \sqrt{3})x^2}{200}$

$\frac{200}{200} - \frac{(2 - \sqrt{3})x^2}{200}$

$\cos PBQ = 1 - \frac{(2 - \sqrt{3})}{200}x^2$ ✓

AQA