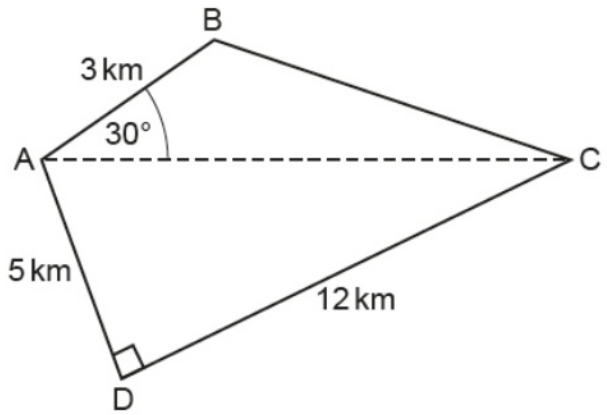
G59...Trigonometry - Area of a Triangle

OCR

Video created by W Neill

20 The diagram shows some land in the shape of a quadrilateral, ABCD.



Not to scale

AB = 3 km, AD = 5 km, CD = 12 km and angle BAC = 30°.

The land is sold for £10 million per square kilometre.

Calculate the total cost of the land.

£ ..... million **[7]**

20 The diagram shows some land in the shape of a quadrilateral, ABCD.

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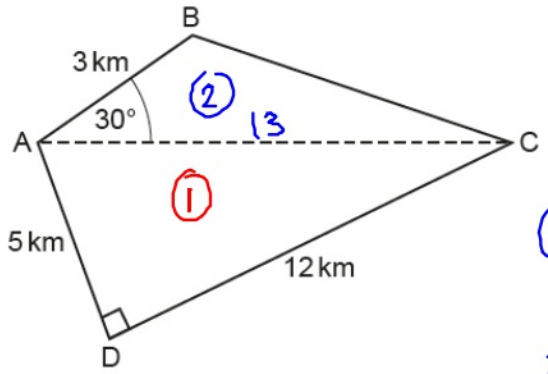
G43
G48
G59



Not to scale

AB = 3 km, AD = 5 km, CD = 12 km and angle BAC = 30°.

The land is sold for £10 million per square kilometre.

Calculate the total cost of the land.

① $\frac{B \times H}{2} = \frac{12 \times 5}{2} = \frac{60}{2}$

$= 30 km^2$

② $\frac{1}{2}ab\sin C$

$AC^2 = 12^2 + 5^2$

$= 144 + 25$

$= 169$

$AC = \sqrt{169} = 13$

Area

$\frac{1}{2}ab\sin C$

$\frac{1}{2}(3)(13)\sin 30°$

$\frac{1}{2}(3)(13)(\frac{1}{2})$

$13 \times 3 = 39$

$\div 2 = 19.5$

$\div 2 = 9.75 km^2$

Total $= 39.75 km^2 \times 10$

397.5 ✓

Sin 30° = $\frac{\sqrt{1}}{2} = \frac{1}{2}$

$= 0.5$

£ ..... million **[7]**

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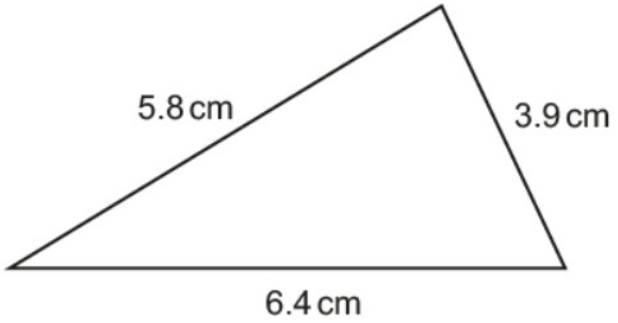
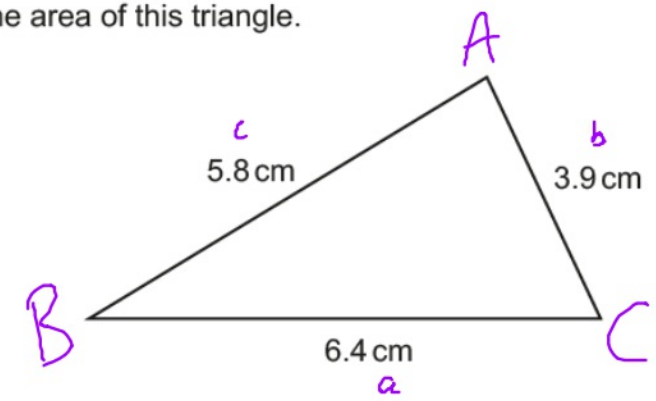
18 Calculate the area of this triangle.



Not to scale

..... cm^2 **[6]**

18 Calculate the area of this triangle.



Not to scale

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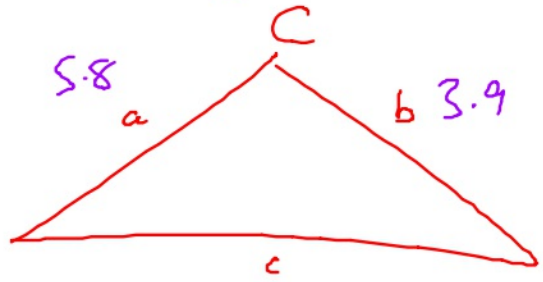
$$\text{Cos}A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{3.9^2 + 5.8^2 - 6.4^2}{2(3.9)(5.8)}$$

$$\text{Cos}A = 0.1746$$

$$A = \text{Cos}^{-1}\, 0.1746$$

$$A = 79.95607107^{\circ}$$

$$\text{Area} = \tfrac{1}{2}ab\,\text{Sin}\,C$$



$$\text{Area} = \tfrac{1}{2}(5.8)(3.9)\,\text{Sin}\,79.95$$

$$= 11.14\,\text{cm}^2$$

✓

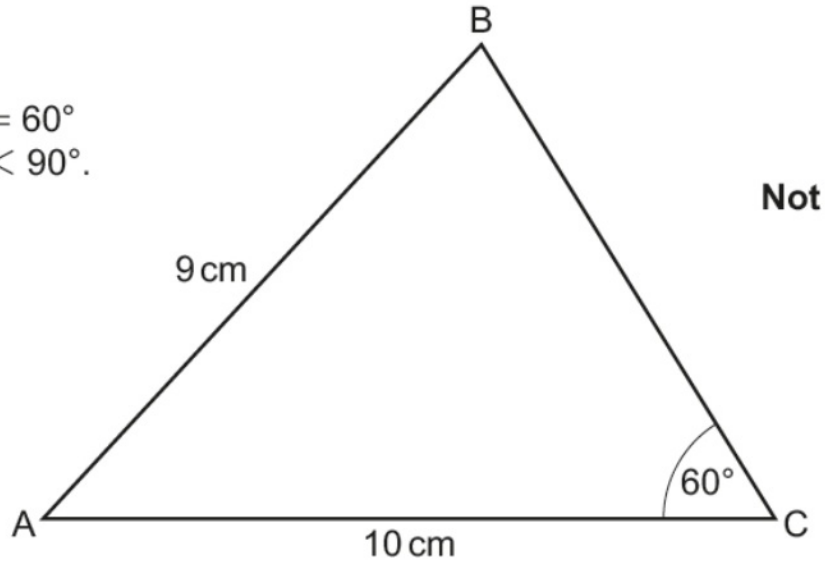
..... cm^2 **[6]**

Video created by W Neill

19 In this triangle:

- AB = 9 cm
- AC = 10 cm
- BC > 5 cm
- angle BCA = 60°
- angle ABC < 90°.

G57
G59



Not to scale

Calculate the area of triangle ABC.

..... cm^2 **[6]**

Video created by W Neill

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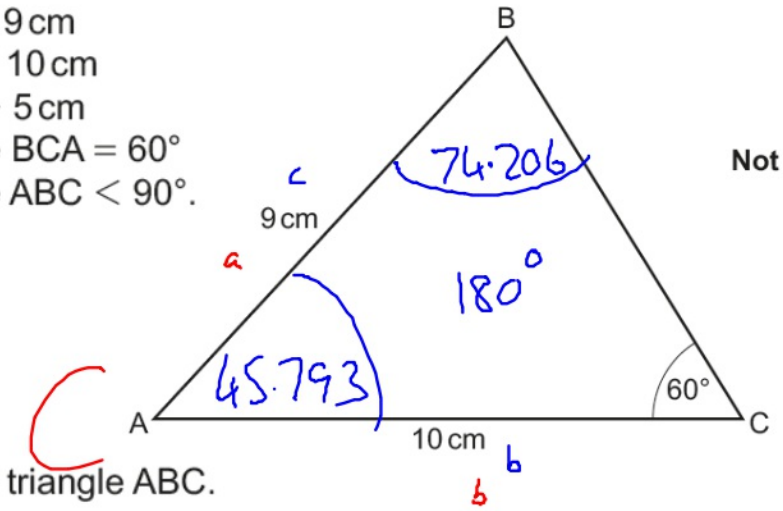

Not to scale

$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin B}{10} = \frac{\sin 60}{9}$ → x

$\sin B = 0.962$

$B = \sin^{-1} 0.962$

$B = 74.206°$

Calculate the area of triangle ABC.

$\frac{1}{2}ab\sin C$

Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}(9)(10)\sin 45.793$

$=$

32.257 cm² ..... cm² **[6]**

Edexcel

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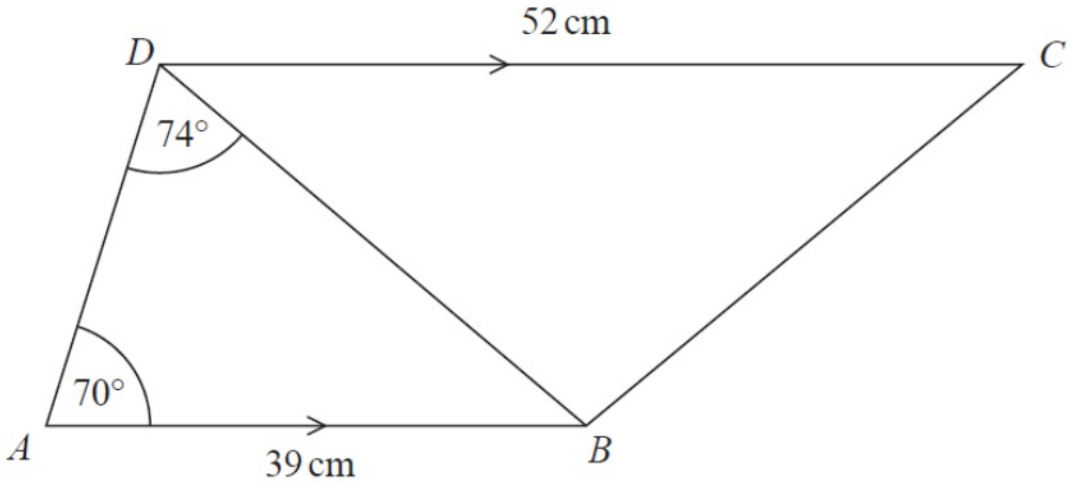
G7
G57
G59

15 Here is trapezium $ABCD$.



AB and DC are parallel.

Work out the area of triangle BCD.
Give your answer correct to 3 significant figures.

..... cm^2

(Total for Question 15 is 5 marks)

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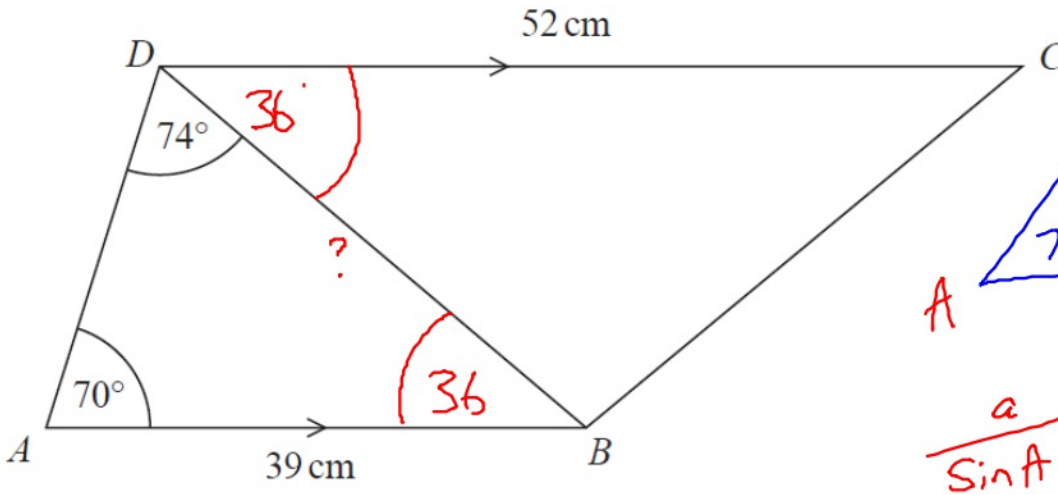
15 Here is trapezium $ABCD$.

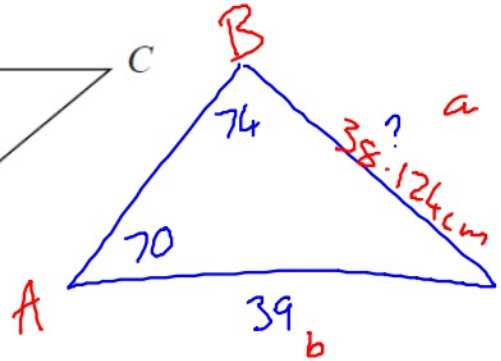
G7
G57
G59



AB and DC are parallel.

Work out the area of triangle BCD.
Give your answer correct to 3 significant figures.

$\frac{1}{2}ab\sin C$

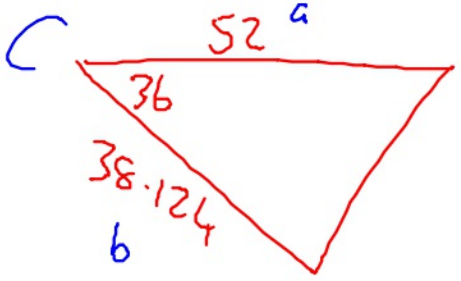


$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$= \frac{a}{\sin 70} = \frac{39}{\sin 74}$$

$$a = 38.124 \text{ cm}$$

Area



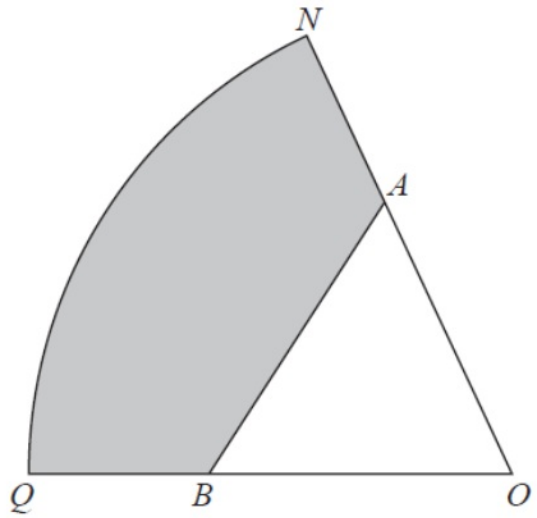
$$= \frac{1}{2}ab\sin C$$

$$\left(\tfrac{1}{2}\right) \times (38.124) \times (52) \times \sin 36 = 583 \text{ cm}^2$$

(Total for Question 15 is 5 marks)

Video created by W Neill

17



ONQ is a sector of a circle with centre *O* and radius 11 cm.

A is the point on *ON* and *B* is the point on *OQ* such that *AOB* is an equilateral triangle of side 7 cm.

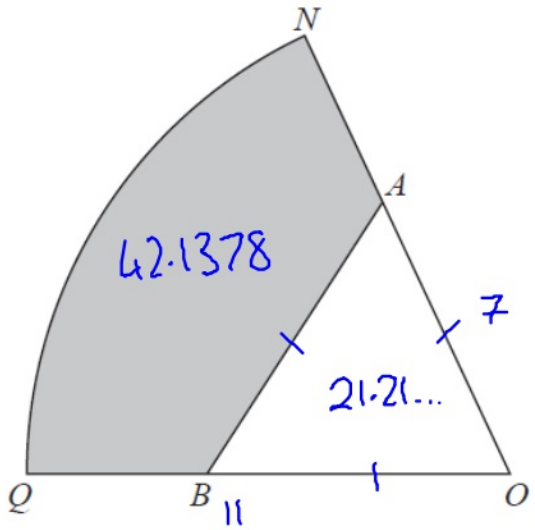
Calculate the area of the shaded region as a percentage of the area of the sector *ONQ*.
Give your answer correct to 1 decimal place.

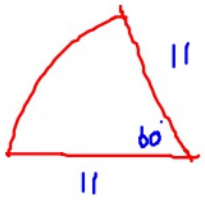
.....%

(Total for Question 17 is 5 marks)

17

Video created by W Neill



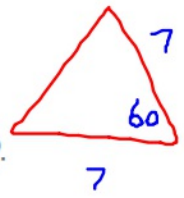


Area ... $R^2 \times \pi$

$11^2 \times \pi \div 6$

$= 63.355545185 cm^2$

Area of $\Delta = \frac{1}{2}ab\sin C$



$= \frac{1}{2}(7)(7)\sin 60$

$= 21.21762239 cm^2$

ONQ is a sector of a circle with centre *O* and radius 11 cm.

A is the point on *ON* and *B* is the point on *OQ* such that *AOB* is an equilateral triangle of side 7 cm.

Calculate the area of the shaded region as a percentage of the area of the sector *ONQ*.
Give your answer correct to 1 decimal place.

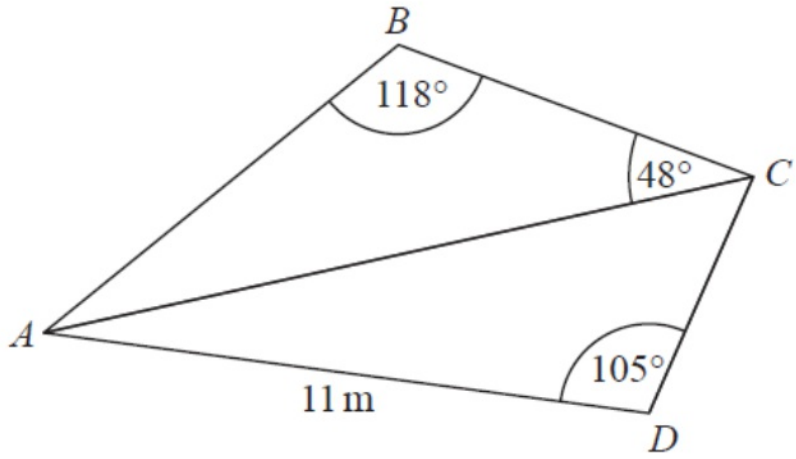
$\frac{42.1378}{63.35545} = 0.6651$

$= 66.51\%$

.....%

(Total for Question 17 is 5 marks)

17 ABC and ADC are triangles.



The area of triangle ADC is $56\,m^2$

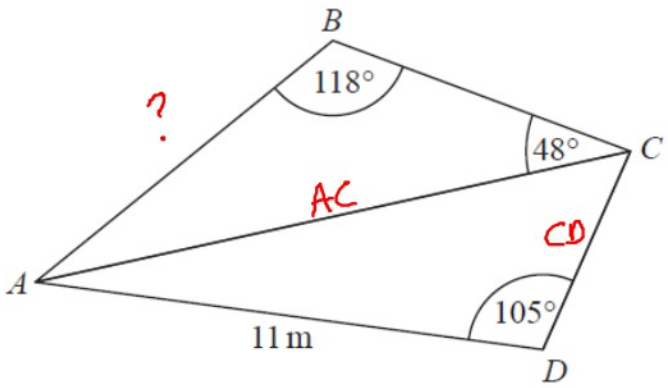
Work out the length of AB.
Give your answer correct to 1 decimal place.

..... m

(Total for Question 17 is 5 marks)

Created by W Neill

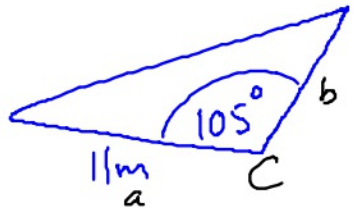
17 *ABC* and *ADC* are triangles.



Angle → opposite Side
Sine Rule

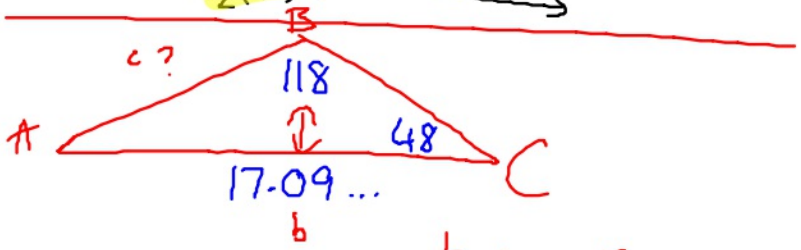
The area of triangle *ADC* is 56 m²

Work out the length of *AB*.
Give your answer correct to 1 decimal place.

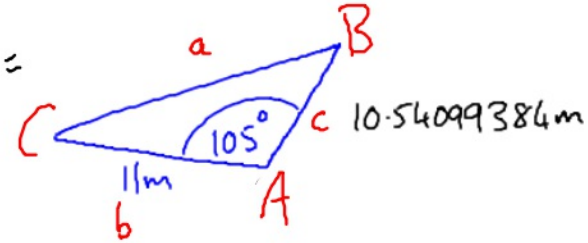


$\frac{1}{2}ab\sin C = 56m^2$

$\frac{1}{2}(11)\,b\,\sin 105 = 56$



AC =



$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{17.09}{\sin 118} = \frac{c}{\sin 48}$

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 232.1125511 - -60.020...$

$a^2 = 292.133$

$a = \sqrt{\ }$

$a = 17.09190365$

14·4 ✓ m

(Total for Question 17 is 5 marks)

AQA

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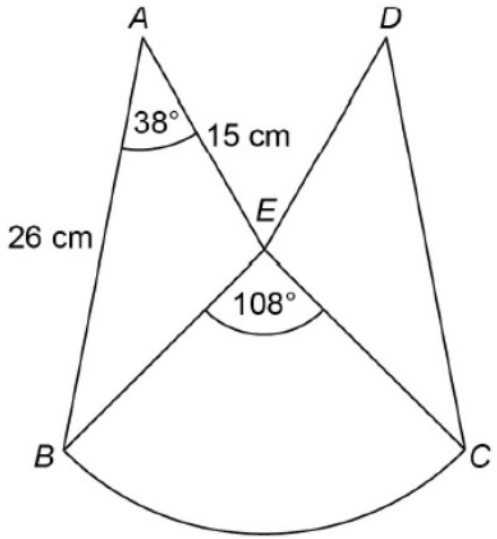
25 The diagram shows a logo.

ABE and *DCE* are congruent triangles.

BCE is a sector of a circle, centre *E*.

G58
G59
G25



Not drawn accurately

Show that the area of the logo is 510 cm^2 to 2 significant figures.

[5 marks]

Video created by W Neill

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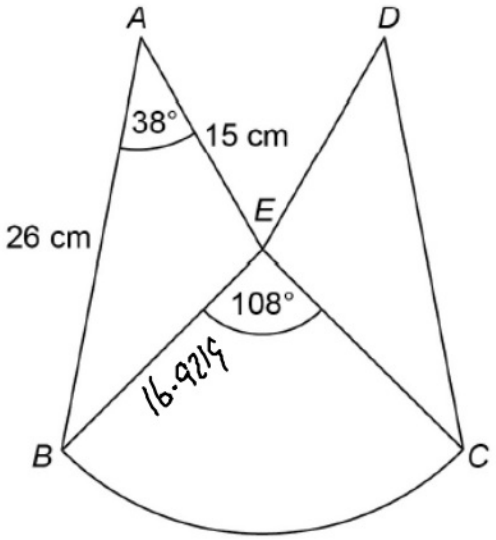
ABE and *DCE* are congruent triangles.

BCE is a sector of a circle, centre *E*.

G58
G59
G25



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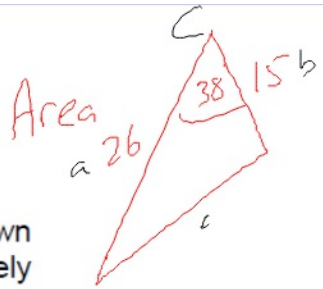
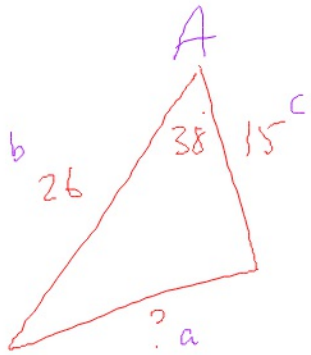


$= \frac{1}{2}ab\sin C = \frac{1}{2}(26)(15)\sin 38$

$120.05\ cm^2$

2 triangles $= 120.05 \times 2$

$= 240.1079754\ cm^2$



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 26^2 + 15^2 - 2(26)(15)\cos 38$

$a^2 = 901 - 614.66$

$a^2 = 286\ldots$

$a = \sqrt{286}$

$a = 16.9219\ cm$

Show that the area of the logo is 510 cm^2 to 2 significant figures.

Area of sector $= R^2 \times \pi \div 360 \times 108$

$= 269.879\ cm^2$

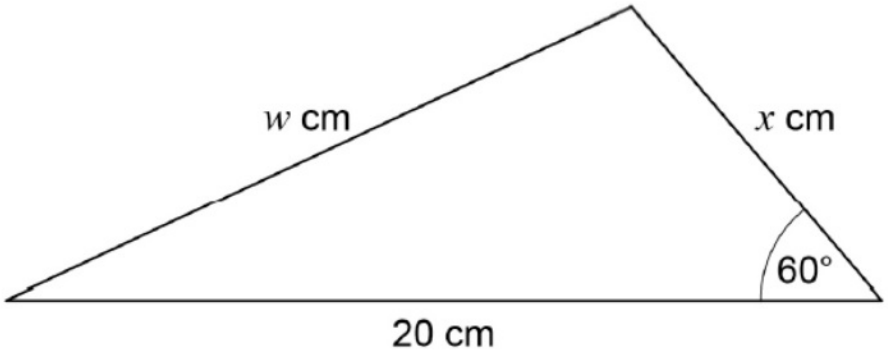
Total $= 240\ldots + 269\ldots$

$= 509.9\ cm^2 = 510\ cm^2$

26 The area of this triangle is $25\sqrt{3}$ cm^2

G48
G58
G59

Not drawn accurately



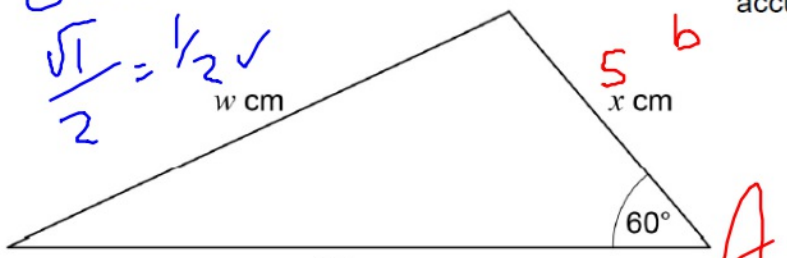
Work out the value of w.
Give your answer in the form $a\sqrt{b}$ where a and b are integers greater than 1

[5 marks]

Answer _____

26 The area of this triangle is $25\sqrt{3}$ cm^2

G48
G58
G59



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Answer _____