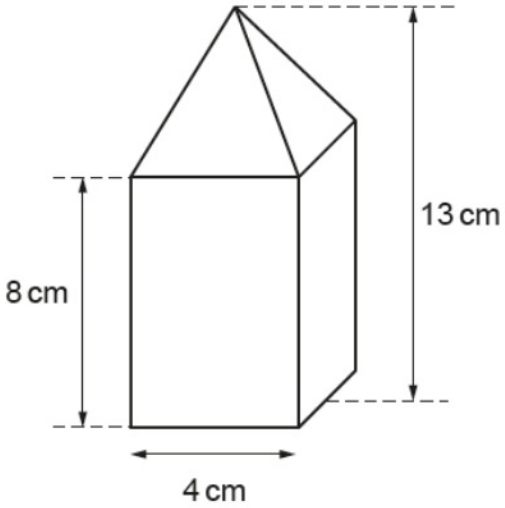
G34 - Volumes of Pyramids, Cones & Spheres

OCR

Video created by W Neill

8 The object below is made from a square-based pyramid joined to a cuboid.



The base of the cuboid and the base of the pyramid are both squares of side 4 cm.
The height of the cuboid is 8 cm and the total height of the object is 13 cm.
The total mass of the object is 158 g.

The cuboid is made from wood with density 0.67 g/cm^3.
The pyramid is made from granite.

Calculate the density of the granite.

[The volume of a pyramid is $\frac{1}{3}$ × area of base × perpendicular height.]

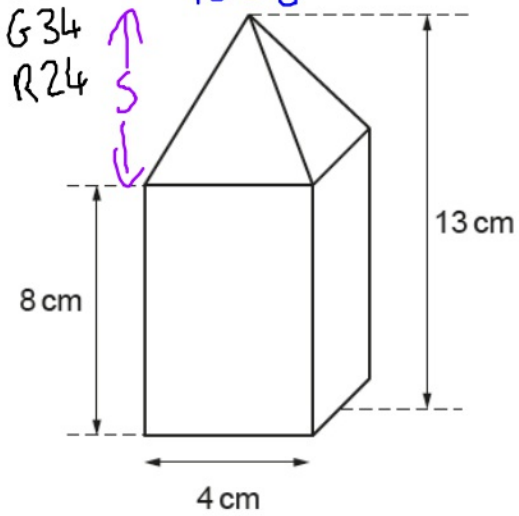
..... g/cm^3 **[5]**

Video created by W Neill

8 The object below is made from a square-based pyramid joined to a cuboid.

G31
G34
R24

158g



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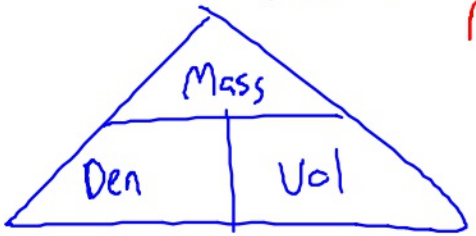
The total mass of the object is 158 g.

The cuboid is made from wood with density 0.67 g/cm^3.
The pyramid is made from granite.

Calculate the density of the granite.

[The volume of a pyramid is $\frac{1}{3}$ × area of base × perpendicular height.]

Wood → 4 x 4 x 8
= 128 cm^3



Mass = 0.67 x 128

85.76g

granite = mass = 72.24g

Volume = 1/3 x 4 x 4 x 5

= 26.6 cm^3

= D = 72.24 / 26.6

2.709 ..... g/cm^3 **[5]**

15 The following formula is for the area, A, of the curved surface area of a cone.
G34 $A = \pi rl$, where r is the radius and l is the slant height of the cone.

Calculate the **total** surface area of a cone with radius 5 cm and slant height 12 cm.

..... cm^2 **[3]**

15 The following formula is for the area, A, of the curved surface area of a cone.
G34 $A = \pi rl$, where r is the radius and l is the slant height of the cone.

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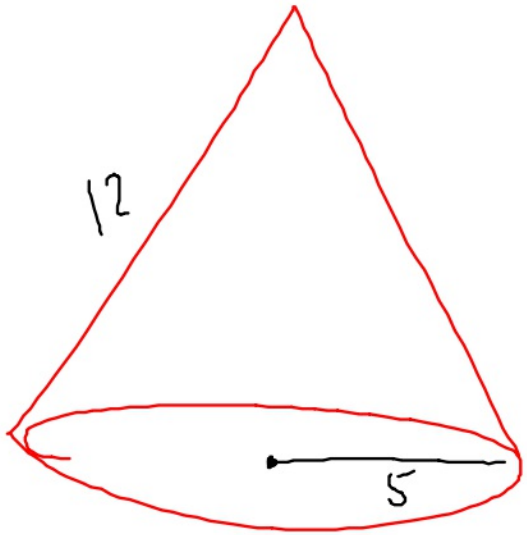

curved face = πrl

$= \pi \times 5 \times 12$

$= 60\pi$

Base = $r^2 \times \pi$

$5^2 \times \pi = 25\pi$ +

267.04 cm^3 / 85π / ..... cm^2 **[3]**

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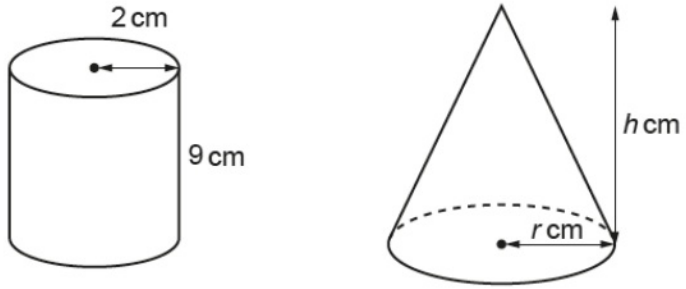
G32
G34

14 The diagram shows a cylinder and a cone.



The cylinder has radius 2 cm and height 9 cm.
The cone has radius r cm and height h cm.

The ratio $r : h$ is 1 : 4.
The volume of the cone is **equal to** the volume of the cylinder.

Work out the value of ***r***.

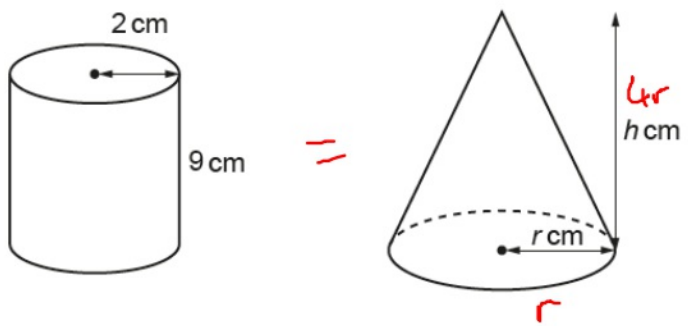
[The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

..... **[5]**

Created by W Neill

14 The diagram shows a cylinder and a cone.

G32
G34



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The cone has radius r cm and height h cm.

The ratio r : h is 1 : 4.
The volume of the cone is **equal to** the volume of the cylinder.

Work out the value of r.

[The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Volume of cylinder
$R^2 \times \pi \times 9$
$= 2^2 \times \pi \times 9$
$= 4 \times \pi \times 9$
$= 36\pi$

$r^2 \times 4r$
$= 4r^3$

$\frac{1}{3}\pi r^2 4r = 36\pi$

$\frac{1}{3} 4r^3 = 36$

$\frac{1}{3} r^3 = 9$ (÷4)

$r^3 = 27$ (x3)

$\sqrt[3]{27} = 3$

$r = 3$ ✓

..... **[5]**

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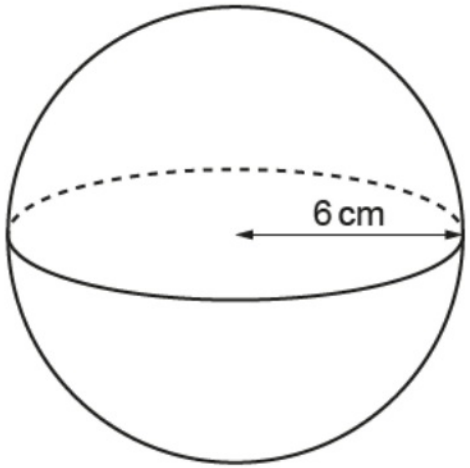
13 (a) Calculate the volume of a sphere with radius 6 cm.



[The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

(a) ..... cm^3 **[2]**

(b) An ornament is made from a solid glass square-based pyramid.
The base has side length 15 cm.
A hemisphere with radius 6 cm is cut out of the base of the pyramid.
This reduces the volume of glass contained in the ornament by 30%.

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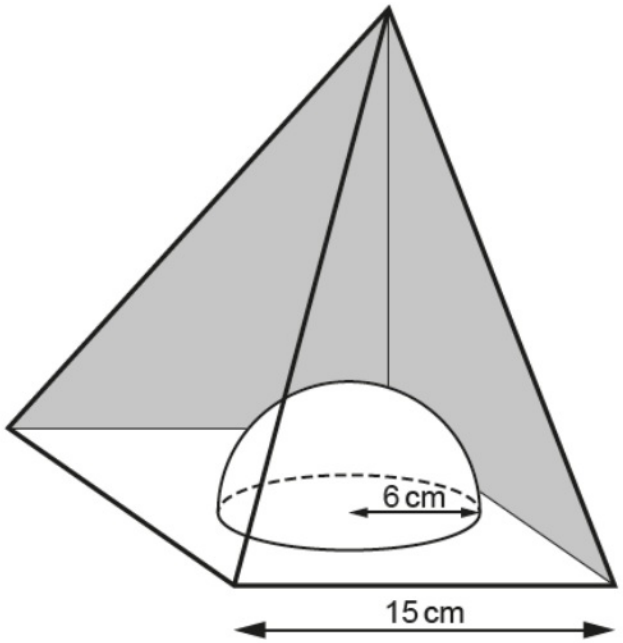

Calculate the perpendicular height of the pyramid.

[The volume of a pyramid is $\frac{1}{3}$ × area of base × perpendicular height.

A hemisphere is half a sphere.]

(b) ..... cm **[5]**

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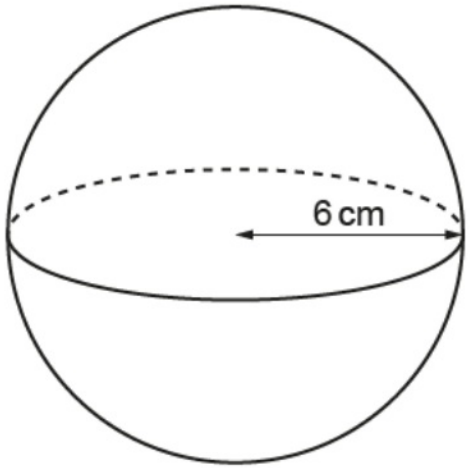
13 (a) Calculate the volume of a sphere with radius 6 cm.

G34



$V = \frac{4}{3}\pi 6^3$

$=$

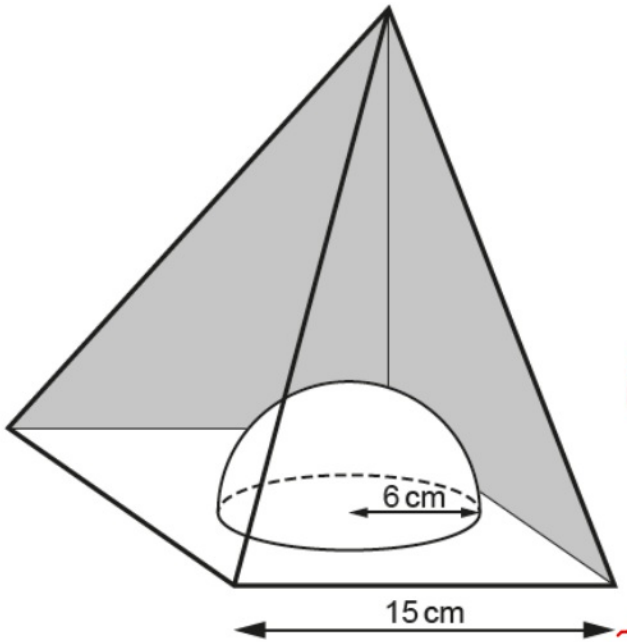
[The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

(a) 288π cm^3 **[2]**

(b) An ornament is made from a solid glass square-based pyramid.
The base has side length 15 cm.
A hemisphere with radius 6 cm is cut out of the base of the pyramid.
This reduces the volume of glass contained in the ornament by 30%.

G35

Created by W Neill



Calculate the perpendicular height of the pyramid.

[The volume of a pyramid is $\frac{1}{3}$ × area of base × perpendicular height.

A hemisphere is half a sphere.]

Sphere = 288π

hemisphere = 144π = 30%

Full shape hemi-s = 144π = 30%.

48π = 10%.

480π = = 100%.

Full pyramid

$\frac{1}{3} \times 15 \times 15 \times h = 480\pi$

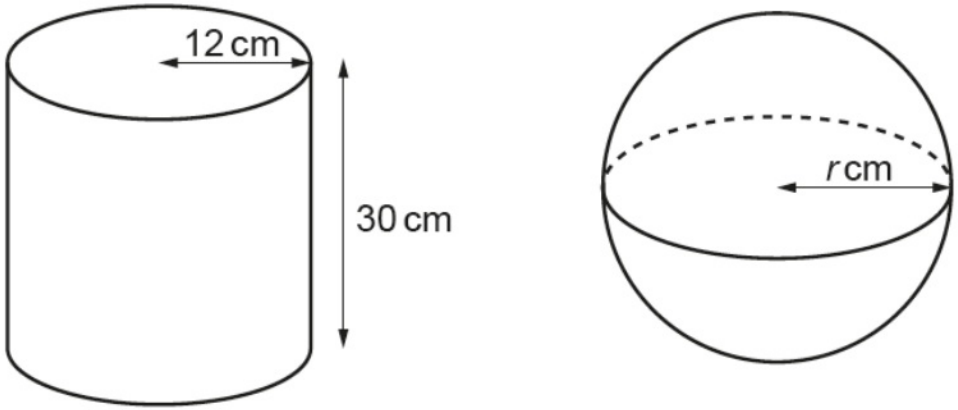
$75h = 480\pi$

$h = \frac{480\pi}{75} =$

(b) 20.106 ✓ cm **[5]**

6 The diagram shows a cylinder and a sphere.

G32
G34



The cylinder has radius 12 cm and height 30 cm.
The cylinder and the sphere have the same volume.

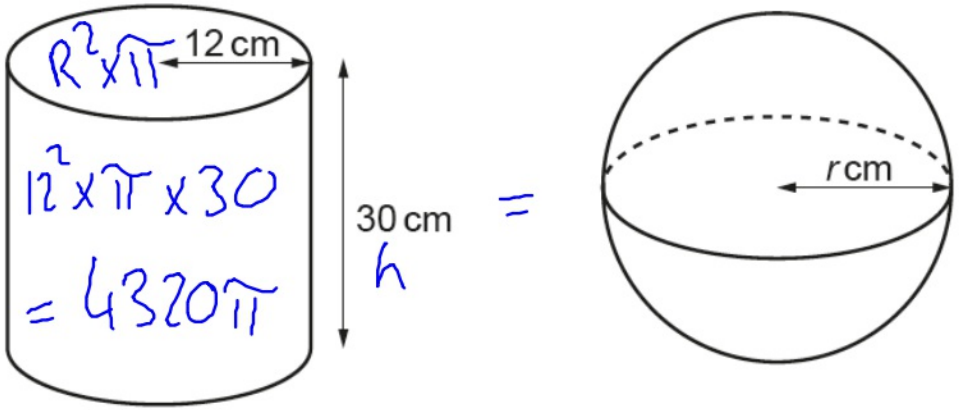
Work out the radius r cm of the sphere.

[The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

..... cm **[5]**

6 The diagram shows a cylinder and a sphere.

G32
G34



$R^2 \times \pi$

$12^2 \times \pi \times 30$

$= 4320\pi$

h

=

$\frac{4}{3}\pi r^3 = 4320\pi$

$\frac{4}{3}r^3 = 4320$

$r^3 = \frac{4320}{4/3}$

$r^3 = 3240$

$r = \sqrt[3]{3240}$

The cylinder has radius 12 cm and height 30 cm.
The cylinder and the sphere have the same volume.

Work out the radius r cm of the sphere.

[The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

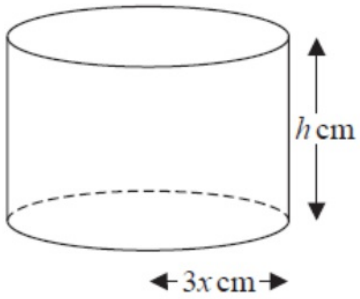
14.8 cm ✓

..... cm **[5]**

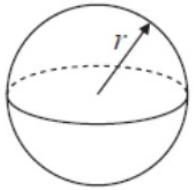
Edexcel

Video created by W Neill

17 The diagram shows a solid metal cylinder.



Volume of sphere $= \frac{4}{3}\pi r^3$



The cylinder has base radius $3x$ cm and height h cm.

The metal cylinder is melted.
All the metal is then used to make 270 spheres.

Each sphere has a radius of $\frac{1}{2}x$ cm.

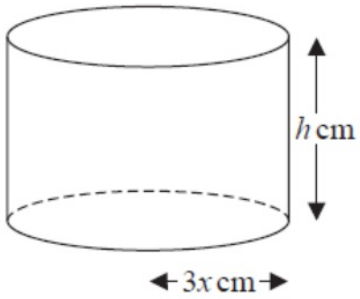
Find an expression, in its simplest form, for h in terms of x.

.....

(Total for Question 17 is 3 marks)

17 The diagram shows a solid metal cylinder.

Video created by W Neill



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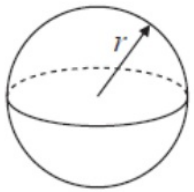


$(3x)^2$ .. $3x \times 3x = 9x^2$

$(\frac{1}{2}x)^3$ ... $\frac{1}{2}x \times \frac{1}{2}x \times \frac{1}{2}x = \frac{1}{8}x^3$

Cylinder

270 spheres

The cylinder has base radius $3x$ cm and height h cm.

The metal cylinder is melted.
All the metal is then used to make 270 spheres.

Each sphere has a radius of $\frac{1}{2}x$ cm.

Find an expression, in its simplest form, for h in terms of x.

$r^2 \times \pi \times h = \frac{4}{3}\pi r^3 \times 270$

$(3x)^2 \pi h = \frac{4}{3}\pi(\frac{1}{2}x)^3 270$

$9x^2 \pi h = \frac{4}{3}\pi \frac{1}{8}x^3 (270)$

$\frac{4}{3} \times \frac{1}{8} \times x^3 \times 270$

$9x^2 h = 45x^3$

$h = \frac{45x^3}{9x^2}$

$h = 5x^1$

(Total for Question 17 is 3 marks)

Video created by W Neill

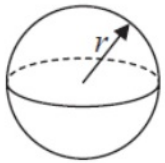
14 Jan has some metal that she is going to make into solid metal spheres.

Each sphere will have a radius of 2.15 cm.

Jan has 1490 cm^3 of metal.

(a) Work out an estimate for the number of spheres that Jan can make.

Volume of sphere $= \frac{4}{3}\pi r^3$



.....

(3)

(b) If you calculate the number of spheres accurately, how do you think your answer to part (a) will change?
Give a reason for your answer.

.....

.....

Video created by W Neill

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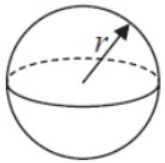


2cm

1500 cm³ (2sf)

(a) Work out an estimate for the number of spheres that Jan can make.

1SF

1 sphere $\frac{4}{3}\pi r^3$

3·14 ... 3 to 1sf

$\frac{4}{3} \times 3 \times 2^3$

$= \frac{4}{3} \times 3 \times 8$

$= \frac{4}{3} \times 24$

$= 32\text{cm}^3$

1 sphere = 32cm^3

$\approx 30\text{cm}^3$

Spheres = $\frac{1500}{30} = 50$ spheres

50 ✓

(3)

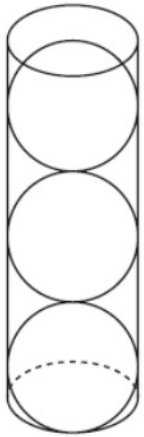
(b) If you calculate the number of spheres accurately, how do you think your answer to part (a) will change?
Give a reason for your answer.

A sphere will be bigger than 30cm^3 (Rounded down 3 times)

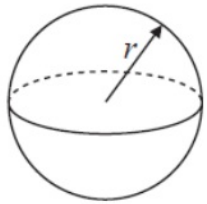
If sphere is bigger, you would have less of them.

Video created by W Neill

8 A hollow cylinder has radius r cm and height $6r$ cm.
3 spheres, also of radius r cm, are put into the cylinder.



Volume of sphere $= \frac{4}{3}\pi r^3$



(a) Work out the proportion of the cylinder that is **not** filled by the spheres.

.....

(3)

Video created by W Neill

The height of the cylinder is increased by $2r$ cm.
Another sphere of radius r cm is put into the cylinder.

Malcolm says,

"There is no change in the proportion of the cylinder **not** filled by the spheres."

(b) Is Malcolm correct?
Justify your answer.

.....

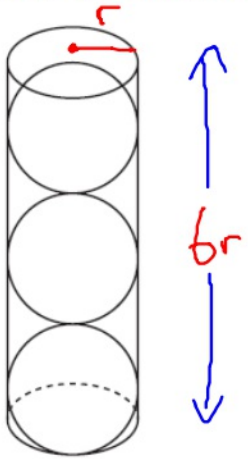
.....

(1)

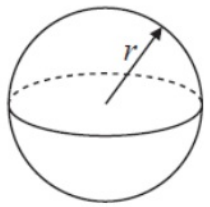
(Total for Question 8 is 4 marks)

Video created by W Neill

8 A hollow cylinder has radius r cm and height $6r$ cm.
3 spheres, also of radius r cm, are put into the cylinder.



Volume of sphere $= \frac{4}{3}\pi r^3$



1 sphere

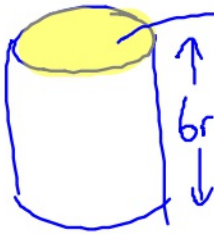
$\frac{4}{3}\pi r^3$

3 spheres

$\frac{3}{1} \times \frac{4}{3}\pi r^3 = \frac{12}{3}\pi r^3 = 4\pi r^3$

(a) Work out the proportion of the cylinder that is **not** filled by the spheres.

Volume of cylinder



$\pi r^2 \times 6r^1$

$= 6\pi r^3$

Cylinder $= 6\pi r^3$

3 spheres $= 4\pi r^3$

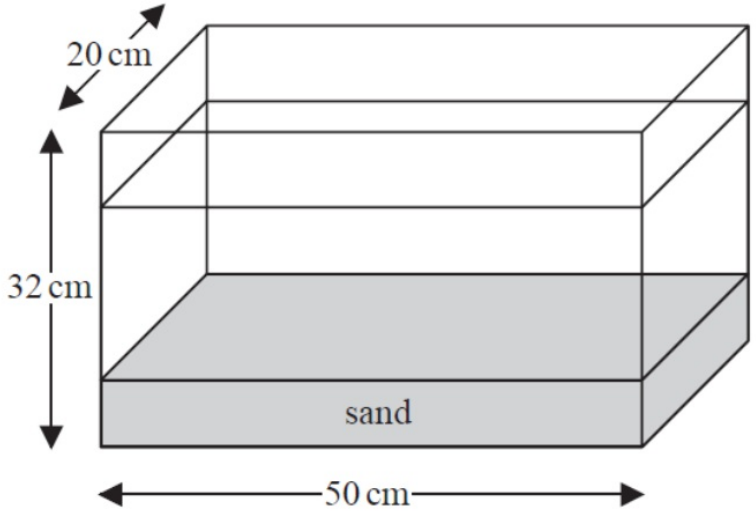
Space left $= 2\pi r^3$

$\frac{2\pi r^3}{6\pi r^3} = \frac{2}{6}$

$\frac{1}{3}$

(3)

2 The diagram shows a fish tank in the shape of a cuboid.



The dimensions of the tank are 50 cm by 32 cm by 20 cm.

The tank is $\frac{3}{4}$ full of water and sand.

The ratio of the volume of water to the volume of sand is 5 : 1

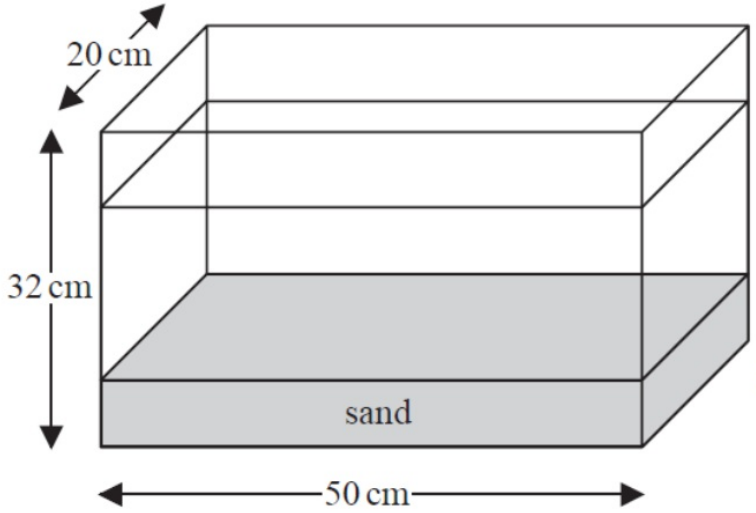
Work out the number of litres of water in the tank.
You must show all your working.

.....litres

(Total for Question 2 is 5 marks)

Created by W Neill

2 The diagram shows a fish tank in the shape of a cuboid.



Volume of cuboid

$50 \times 20 \times 32 = 1000 \times 32$

$= 32000 cm^3$

$\frac{3}{4}$ of $32000 cm^3 = 24000 cm^3$

÷4 ×3

Water : Sand

5 : 1

The dimensions of the tank are 50 cm by 32 cm by 20 cm.

The tank is $\frac{3}{4}$ full of water and sand.

The ratio of the volume of water to the volume of sand is 5 : 1

24000 ÷6

=4000 ×5 ... 20000 cm^3 ← water

4000 ×1 = 4000 cm^3

Work out the number of litres of water in the tank.
You must show all your working.

1000 cm^3 = 1 Litre

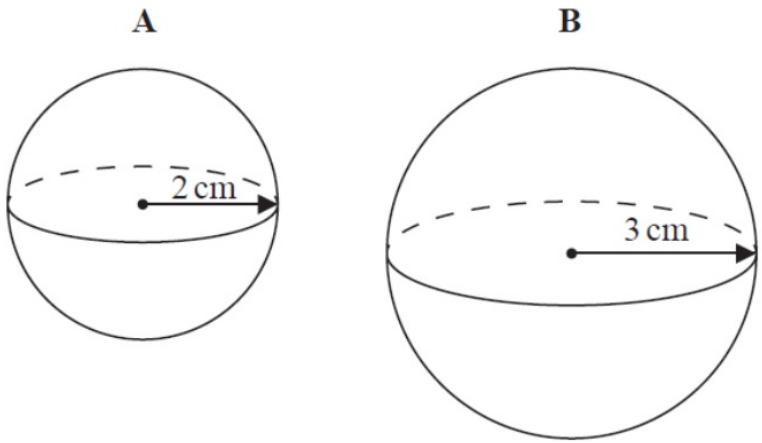
20,000 cm^3 =

20 litres

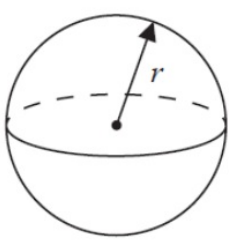
(Total for Question 2 is 5 marks)

Created by W Neill

16 Here are two solid spheres, **A** and **B**.



Volume of sphere $= \frac{4}{3}\pi r^3$



Sphere **A** is made of gold.
Sphere **B** is made of silver.

Sphere **A** has radius 2 cm.
Sphere **B** has radius 3 cm.

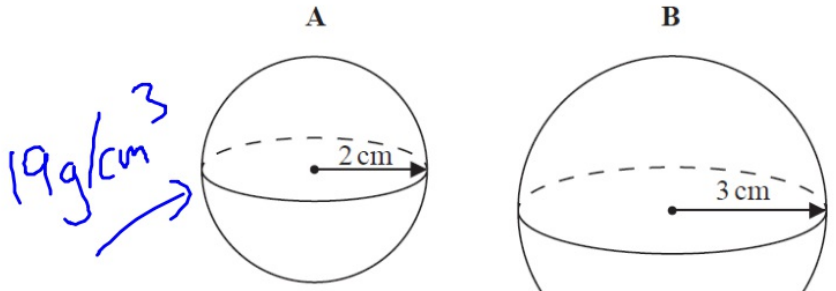
Gold has a density of 19 000 kg/m^3
Silver has a density of 10 000 kg/m^3

Which sphere has the greater mass?
You must show how you get your answer.

(Total for Question 16 is 4 marks)

Created by W Neill

16 Here are two solid spheres, **A** and **B**.

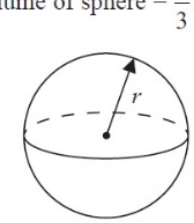


19g/cm³

10g/cm³

Mass | Density | Volume

Volume of sphere = $\frac{4}{3}\pi r^3$



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Gold has a density of 19 000 kg/m³
Silver has a density of 10 000 kg/m³

Which sphere has the greater mass?
You must show how you get your answer.

Gold

$\frac{4}{3}\pi r^3$

$\frac{4}{3} \times \pi \times 8 \times 19$

$= \frac{4}{3}\pi \, 152$

Silver

$\frac{4}{3}\pi \, 3^3 \times 10$

$\frac{4}{3}\pi \, 27 \times 10$

$\frac{4}{3}\pi \, 270$

19000 kg /m³
19000000 g/m³
19g/cm³ ✓

1,000,000cm³ = 1m³

19 × 8
152

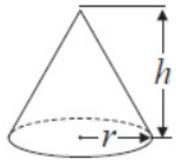
Silver has larger mass ✓
270 > 152

(Total for Question 16 is 4 marks)

15 A cone has a volume of 98 cm^3.
The radius of the cone is 5.13 cm.

(a) Work out an estimate for the height of the cone.

Volume of cone $= \frac{1}{3}\pi r^2 h$



Video created by W Neill

John uses a calculator to work out the height of the cone to 2 decimal places.

.....cm

(3)

(b) Will your estimate be more than John's answer or less than John's answer?
Give reasons for your answer.

.....

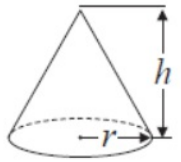
.....

.....

(1)

15 A cone has a volume of 98 cm^3.
The radius of the cone is 5.13 cm.

Volume of cone $= \frac{1}{3}\pi r^2 h$



Video created by W Neill

(a) Work out an estimate for the height of the cone.

1sf $98cm^3 = 100cm^3$

$5.13cm = 5cm$

$\pi = 3.142 = 3$

$\frac{1}{3}\pi r^2 h = 100$

$\frac{1}{3} \times 3 = 1$

$\frac{1}{3} \times 3 \times 5^2 \times h = 100$

$25h = 100$

$25h = 100$

$h = \frac{100}{25}$

$h = 4$

John uses a calculator to work out the height of the cone to 2 decimal places.

4 cm

(3)

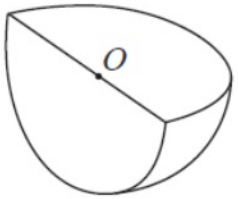
(b) Will your estimate be more than John's answer or less than John's answer?
Give reasons for your answer.

$\frac{1}{3} \times 3.142 \times 5.13^2$ $\frac{98}{>25} < 4$ $\frac{100}{25} = 4$ .... my answer will be bigger than Johns.

(1)

19 Shape **S** is one quarter of a solid sphere, centre O.

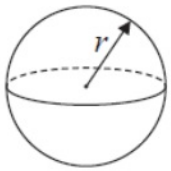
G34
G35



Shape **S**

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



The volume of **S** is 576π cm^3

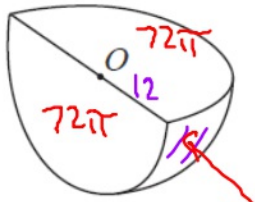
Find the surface area of **S**.
Give your answer correct to 3 significant figures.
You must show your working.

..... cm^2

(Total for Question 19 is 5 marks)

19 Shape **S** is one quarter of a solid sphere, centre O.

G34
G35

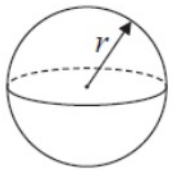


Shape **S**

1/4 of sphere

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



576π x 4

full sphere $= \frac{4}{3}\pi r^3 = 2304\pi$

$\frac{4}{3}r^3 = 2304$

$4r^3 = 2304 \times 3$

$4r^3 = 6912$

$r^3 = 1728 \div 4$

$r = \sqrt[3]{1728}$

$r = 12$

The volume of **S** is 576π cm^3

Find the surface area of **S**.
Give your answer correct to 3 significant figures.
You must show your working.

SA = full sphere $= 4\pi r^2$

$= \frac{1}{4} \times 4\pi \times 12^2 \div 4$

$12^2\pi = 144\pi$

½ Circle $= \frac{r^2 \times \pi}{2} = \frac{144\pi}{2} = 72\pi \times 2$

$= 144\pi$

ans
144π
$+ 144\pi$
$= 288\pi$
$= 904.77$
905 cm^2

(Total for Question 19 is 5 marks)

AQA

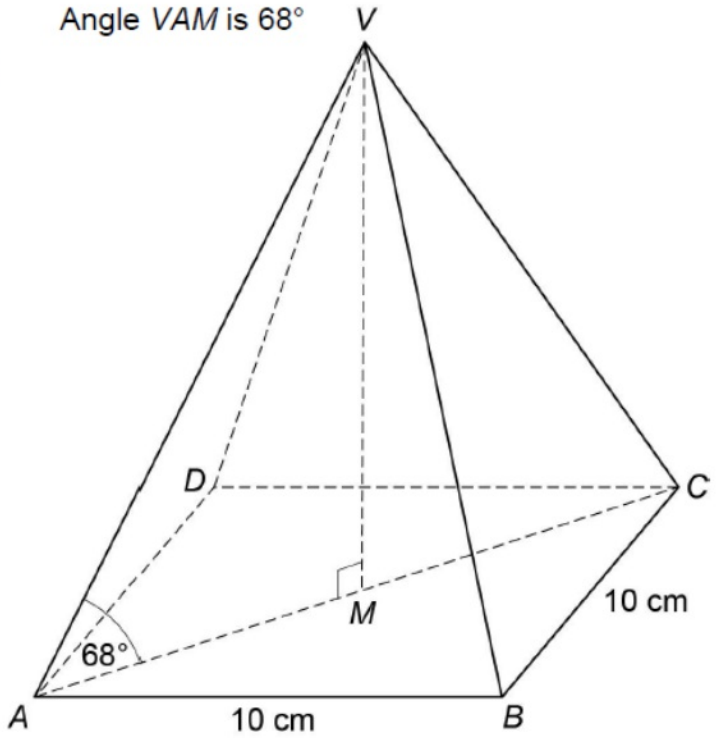
27 *VABCD* is a square-based pyramid.

The horizontal base *ABCD* has side length 10 cm and centre *M*.

G34 Angle *VMA* is 90°

G61 Angle *VAM* is 68°

G62



Work out the volume of the pyramid. **[6 marks]**

Volume of pyramid = $\frac{1}{3}$ × area of base × perpendicular height

Answer _____ cm^3

27 *VABCD* is a square-based pyramid.

The horizontal base *ABCD* has side length 10 cm and centre *M*.

Angle *VMA* is 90°

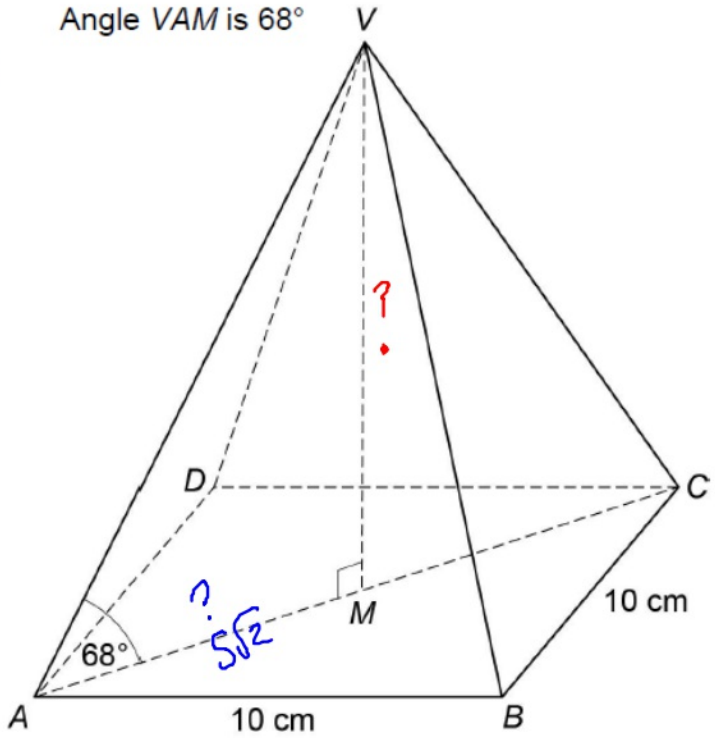
Angle *VAM* is 68°

Work out the volume of the pyramid. **[6 marks]**

G34

G61

G62



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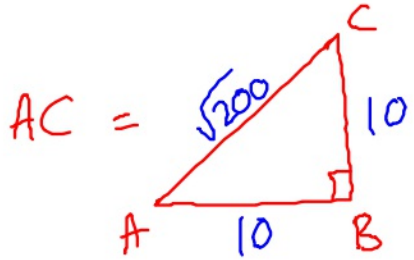
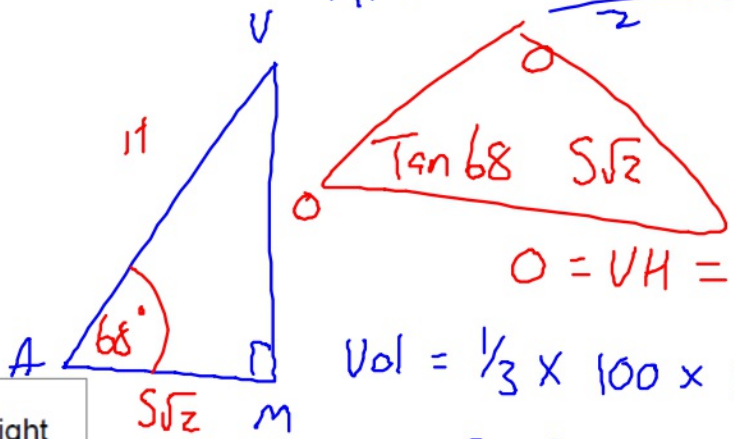


$10^2 + 10^2 = AC^2$

$200 = AC^2$

$\sqrt{200} = AC$

$AM = \frac{\sqrt{200}}{2} = 5\sqrt{2}$



$O = VH = 17.50\text{ cm}$

$Vol = \frac{1}{3} \times 100 \times 17.50\ldots$

Answer 583.38 ✓ cm^3

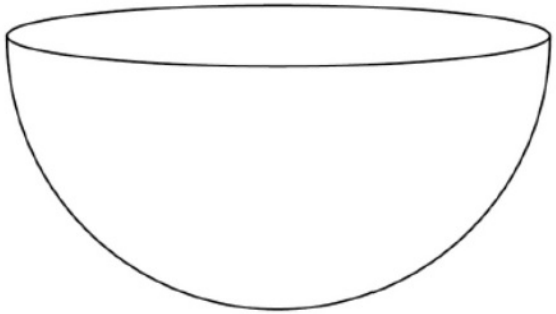
10

Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

Video created by W Neill

G34

G35 A container is a hemisphere of radius 30 cm



Sand fills the container at a rate of 4000 cm^3 per minute.

Does it take **less than** a quarter of an hour to fill the container?

You **must** show your working.

[3 marks]

Answer _____

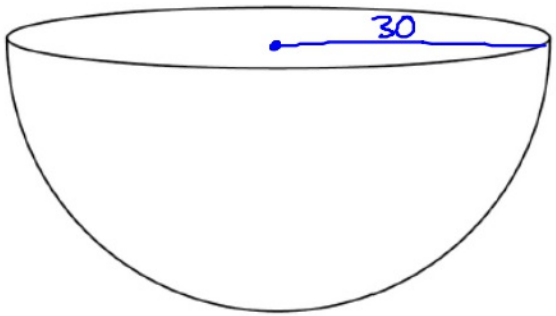
Video created by W Neill

10

Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

G34

G35 A container is a hemisphere of radius 30 cm



Volume full = $\frac{4}{3} \times \pi \times 30^3$

$= 36000\pi$

½ sphere = $\div 2$

$= 56548.66\ cm^3$

Sand fills the container at a rate of 4000 cm^3 per minute.

Does it take **less than** a quarter of an hour to fill the container?

You **must** show your working.

56548... ÷ 4000 = min

Yes it does take less than ¼ hr (15 min) **[3 marks]**

14.13 min

14.13 < 15 ✓

Answer _____

24

G34

Video created by W Neill

Volume of a sphere $= \frac{4}{3}\pi r^3$	where r is the radius
Volume of a cone $= \frac{1}{3}\pi r^2 h$	where r is the radius and h is the perpendicular height

A sphere has radius $2x$ cm

A cone has

radius $3x$ cm

perpendicular height h cm

The sphere and the cone have the same volume.

Work out radius of cone : perpendicular height of cone

Give your answer in the form $a : b$ where a and b are integers. **[4 marks]**

Answer _____ : _____

24

G34

Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

Volume of a cone = $\frac{1}{3}\pi r^2 h$ where r is the radius and h is the perpendicular height

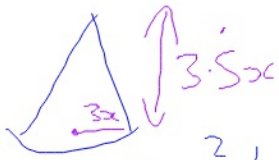
Video created by W Neill

A sphere has radius $2x$ cm

A cone has

radius $3x$ cm

perpendicular height h cm



$\frac{4}{3}\pi(2x)^3 = \frac{1}{3}\pi(3x)^2 h$

$\frac{4}{3}\pi 8x^3 = \frac{1}{3}\pi 9x^2 h$

$\frac{4}{3}\pi 8x^3 = 3\pi x^2 h$

The sphere and the cone have the same volume.

$\frac{4}{3}8x^3 = h$

$3x^2$

$\frac{4}{9}(8x) = h$

$3.5x = h$

Work out radius of cone : perpendicular height of cone

Give your answer in the form $a : b$ where a and b are integers. **[4 marks]**

$3x : 3.5x$

$3 : 3.5$

Answer 27 : 32

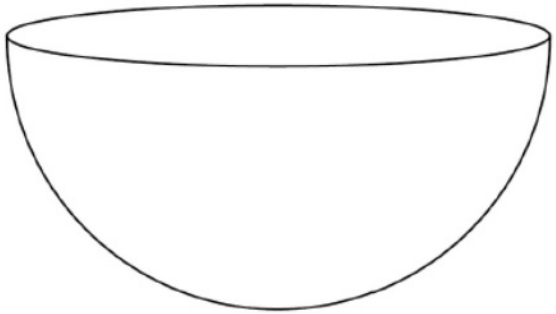
Video created by W Neill

27

Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

G34

G35 A container is a hemisphere of radius 30 cm



Sand fills the container at a rate of 4000 cm^3 per minute.

Does it take **less than** a quarter of an hour to fill the container?

You **must** show your working.

[3 marks]

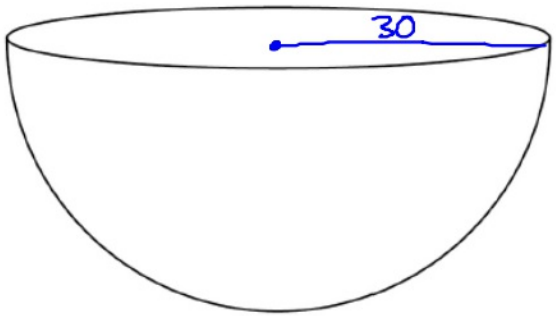
Answer _____

Video created by W Neill

Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

G34

G35 A container is a hemisphere of radius 30 cm



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Does it take **less than** a quarter of an hour to fill the container?

You **must** show your working.

56548... ÷ 4000 = min

Yes it does take less than ¼ hr (15 min) **[3 marks]**

14.13 min

14.13 < 15 ✓

Answer _____

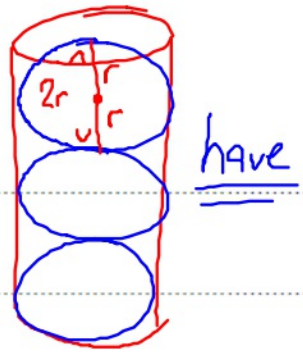
Video created by W Neill

The height of the cylinder is increased by $2r$ cm.
Another sphere of radius r cm is put into the cylinder.

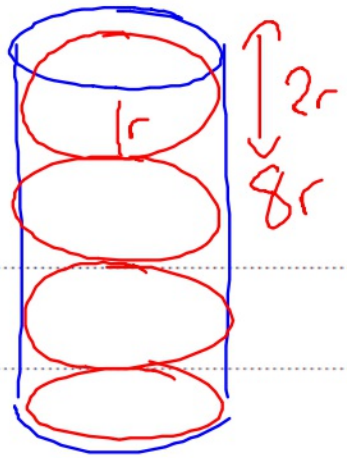
Malcolm says,

"There is no change in the proportion of the cylinder **not** filled by the spheres."

(b) Is Malcolm correct?
Justify your answer.



have



Yes he is correct as the height of cylinder increases by the same amount as the height of the ball added.

(1)

(Total for Question 8 is 4 marks)