G64 (H) Vector Geometry Proof

EDEXCEL

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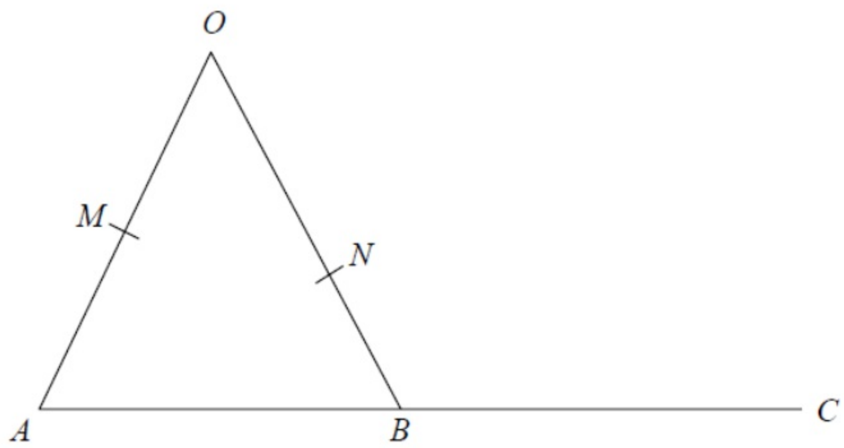
18



OMA, ONB and *ABC* are straight lines.
M is the midpoint of *OA*.
B is the midpoint of *AC*.
$\overrightarrow{OA} = 6\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{ON} = k\mathbf{b}$ where k is a scalar quantity.

Given that *MNC* is a straight line, find the value of k.

(Total for Question 18 is 5 marks)

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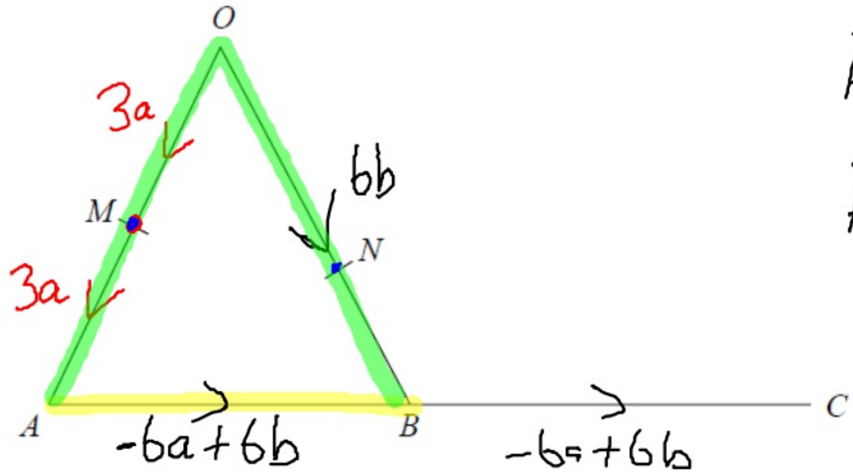
18



OMA, *ONB* and *ABC* are straight lines.
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$\overrightarrow{OA} = 6\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{ON} = k\mathbf{b}$ where k is a scalar quantity.

Given that *MNC* is a straight line, find the value of *k*.

$\overrightarrow{MN}$ $\overrightarrow{MC}$ $\overrightarrow{NC}$

must all be multiples of each other.

$\overrightarrow{AB} = -6a + 6b$

$\overrightarrow{AC} = -12a + 12b$

$\overrightarrow{MC} = 3a - 12a + 12b$
$= -9a + 12b$

$\overrightarrow{MN} = -3a + kB$ (×3)

$K = 4$ ✓

(Total for Question 18 is 5 marks)

22

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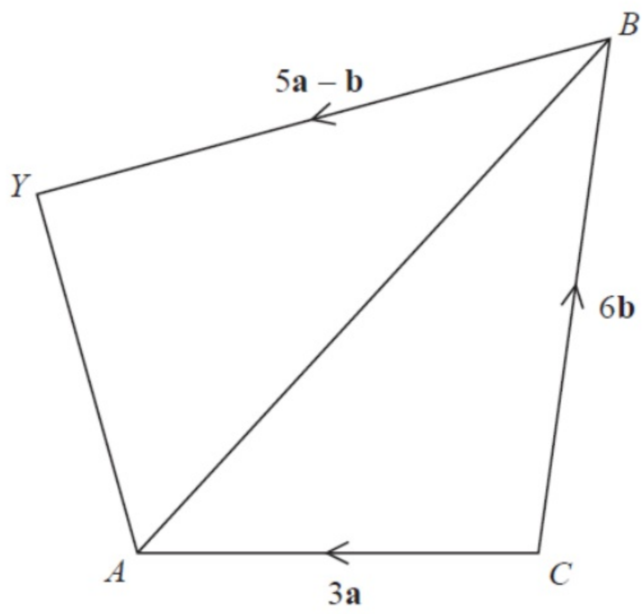

CAYB is a quadrilateral.

$\overrightarrow{CA} = 3\mathbf{a}$
$\overrightarrow{CB} = 6\mathbf{b}$
$\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

X is the point on *AB* such that $AX : XB = 1 : 2$

Prove that $\overrightarrow{CX} = \frac{2}{5}\overrightarrow{CY}$

(Total for Question 22 is 5 marks)

22

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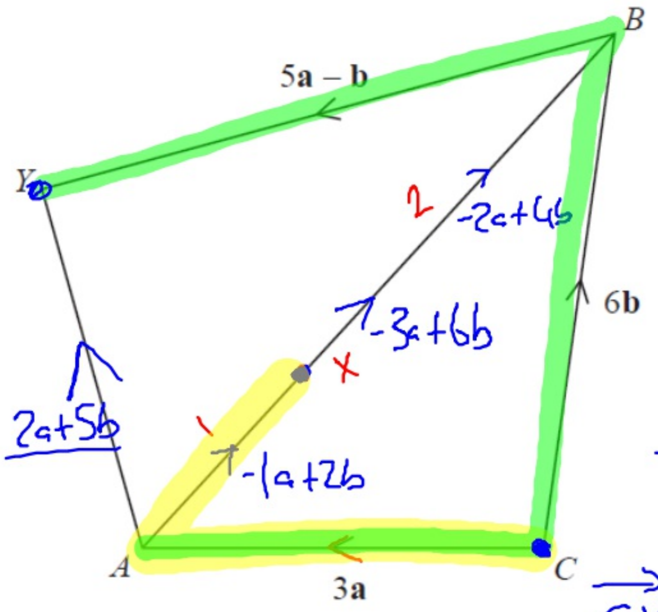

$\overrightarrow{AY}$

$-3a+6b+5a-b$

$2a+5b$

$\overrightarrow{AB} = -3a+6b$

$1 : 2$

$-1a+2b : -2a+4b$

$\overrightarrow{CX} = 3a-1a+2b$

$= 2a+2b$

CX is $\frac{2}{5}$ of CY as 2 is $\frac{2}{5}$ of 5

$\overrightarrow{CY} = 3a+2a+5b$

$= 5a+5b$

Proved

CAYB is a quadrilateral.

$\overrightarrow{CA} = 3\mathbf{a}$

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$\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

X is the point on *AB* such that $AX : XB = 1 : 2$

Prove that $\overrightarrow{CX} = \frac{2}{5}\overrightarrow{CY}$

(Total for Question 22 is 5 marks)

20

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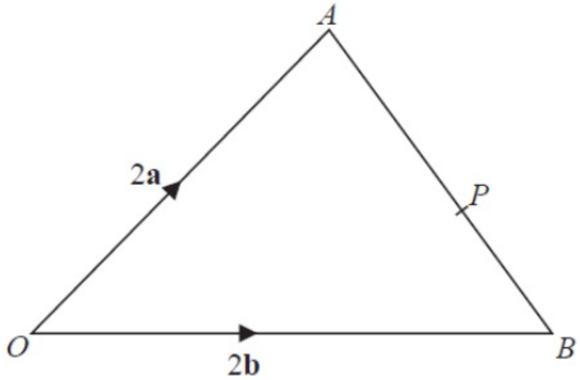

OAB is a triangle.
P is the point on *AB* such that $AP : PB = 5 : 3$

$\overrightarrow{OA} = 2\mathbf{a}$

$\overrightarrow{OB} = 2\mathbf{b}$

$\overrightarrow{OP} = k(3\mathbf{a} + 5\mathbf{b})$ where k is a scalar quantity.

Find the value of k.

....

(Total for Question 20 is 4 marks)

20

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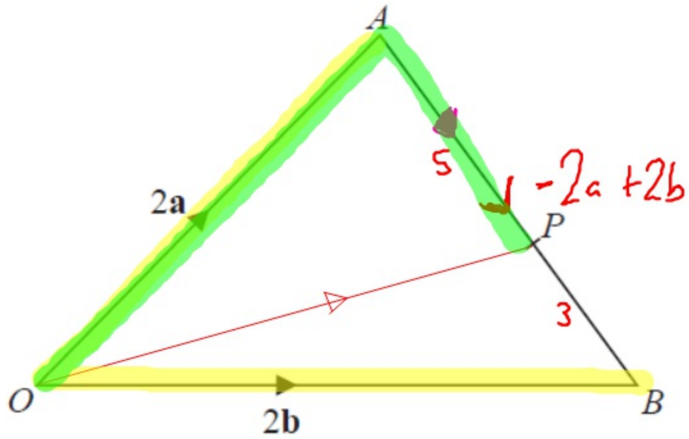

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$\overrightarrow{OA} = 2\mathbf{a}$

$\overrightarrow{OB} = 2\mathbf{b}$

$\overrightarrow{OP} = k(3\mathbf{a} + 5\mathbf{b})$ where k is a scalar quantity.

Find the value of k.

$\overrightarrow{AB} = -2a + 2b$

$\overrightarrow{AP} = -\frac{5}{4}a + \frac{5}{4}b = -1\frac{1}{4}a + 1\frac{1}{4}b$

$\overrightarrow{OP} = 2a - 1\frac{1}{4}a + 1\frac{1}{4}b = \frac{3}{4}a + \frac{5}{4}b$

$\frac{1}{4}(3a + 5b)$

$\frac{1}{4}$

(Total for Question 20 is 4 marks)

16

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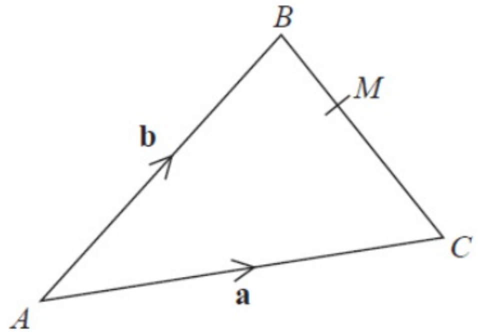

M is the point such that $BM : MC$ is $1 : 2$

Here is Charlie's method to find $\overrightarrow{BM}$ in terms of **a** and **b**.

$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$$
$$= -\mathbf{b} + \mathbf{a}$$
$$= \mathbf{a} - \mathbf{b}$$
$$\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BC}$$
$$= \frac{1}{2}(\mathbf{a} - \mathbf{b})$$

(a) Evaluate Charlie's method.

....

....

(1)

16

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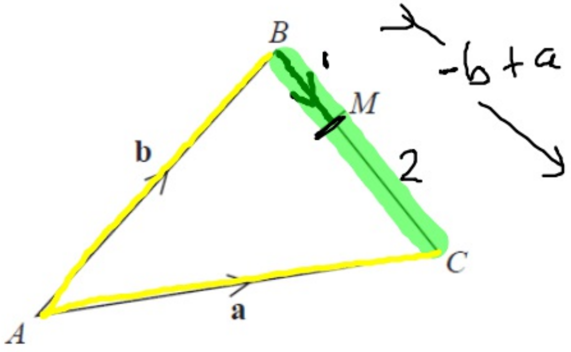

$\overrightarrow{BC} = -b + a$

$= -\frac{1}{3}b + \frac{1}{3}a$ ✓

M is the point such that $BM : MC$ is $1:2$

Here is Charlie's method to find $\overrightarrow{BM}$ in terms of **a** and **b**.

$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$$ ✓

$$= -\mathbf{b} + \mathbf{a}$$ ✓

$$= \mathbf{a} - \mathbf{b}$$ ✓

$$\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BC}$$ → ½ of BC

$$= \frac{1}{2}(\mathbf{a} - \mathbf{b})$$

(a) Evaluate Charlie's method.

Charlie has found ½ of $\overrightarrow{BC}$ to get $\overrightarrow{BM}$

He needed ⅓ of $\overrightarrow{BC}$ to get $\overrightarrow{BM}$

(1)

21

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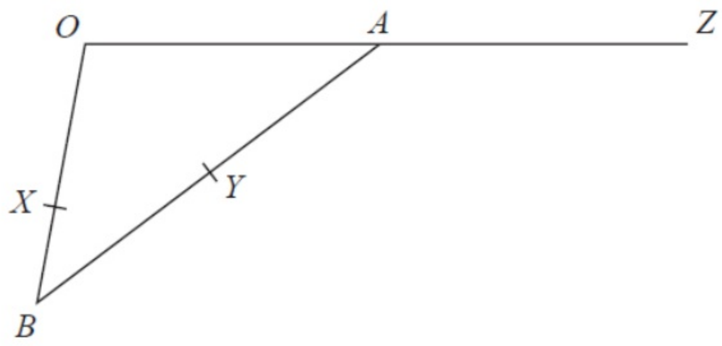

OAB is a triangle.
A is the midpoint of OZ
Y is the midpoint of AB
X is a point on OB

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OX} = 2\mathbf{b}$ $\overrightarrow{XB} = \mathbf{b}$

Prove that XYZ is a straight line.

(Total for Question 21 is 5 marks)

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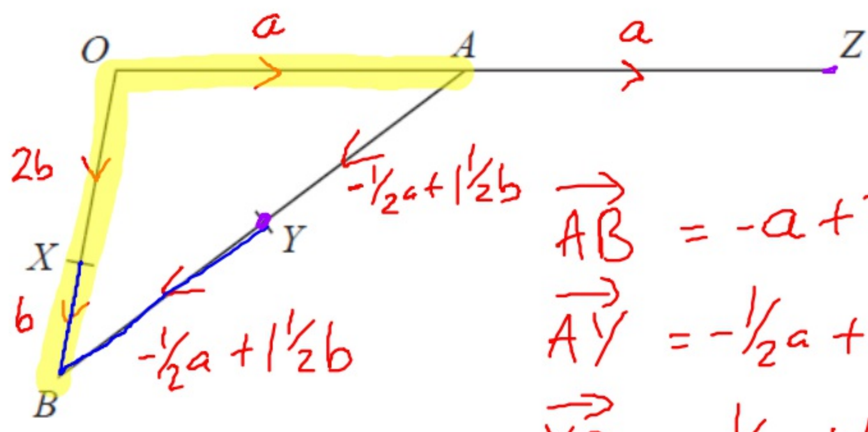
21



$\overrightarrow{AB} = -a + 3b$

$\overrightarrow{AY} = -\frac{1}{2}a + 1\frac{1}{2}b$

$\overrightarrow{YB} = -\frac{1}{2}a + 1\frac{1}{2}b$

lines are parallel and share points they must be straight.

OAB is a triangle.
A is the midpoint of *OZ*
Y is the midpoint of *AB*
X is a point on *OB*

$\overrightarrow{OA} = \mathbf{a}$ $\quad\overrightarrow{OX} = 2\mathbf{b}$ $\quad\overrightarrow{XB} = \mathbf{b}$

Prove that *XYZ* is a straight line.

$\overrightarrow{XY}$ $\quad\overrightarrow{XZ}$ or $\overrightarrow{YZ}$

ans are multiples of each other. parallel.

$\overrightarrow{XY} = +b + \frac{1}{2}a - 1\frac{1}{2}b$

$= \frac{1}{2}a - \frac{1}{2}b$ ✓

$\overrightarrow{YZ} = \frac{1}{2}a - 1\frac{1}{2}b + a$

$= 1\frac{1}{2}a - 1\frac{1}{2}b$ (×3)

$\overrightarrow{XZ} = -2b + 2a$

$2a - 2b$ (×4)

As all of these are multiples, they must be parallel.

(Total for Question 21 is 5 marks)

20

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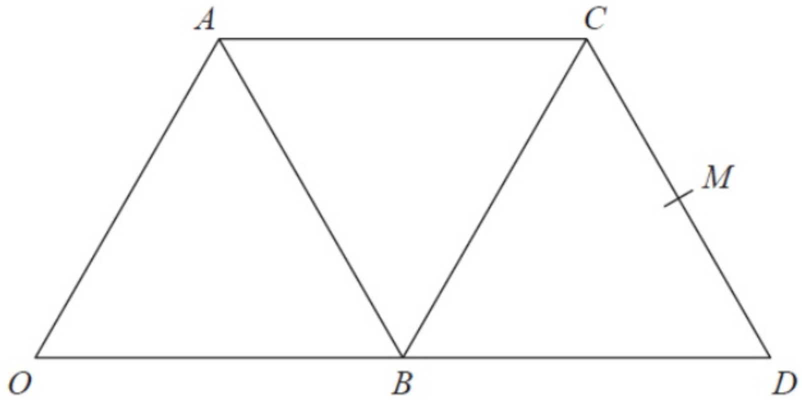

OACD is a trapezium and *OACB* is a parallelogram.
B is the midpoint of *OD*.
M is the midpoint of *CD*.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

Given that $\overrightarrow{BM} = k \times \overrightarrow{OC}$ where k is a scalar,

use a vector method to find the value of k.

....

(Total for Question 20 is 3 marks)

20

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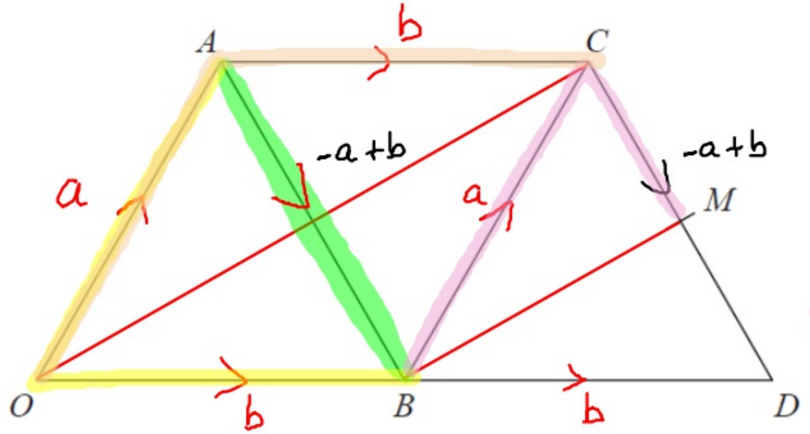

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B is the midpoint of *OD*.
M is the midpoint of *CD*.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

Given that $\overrightarrow{BM} = k \times \overrightarrow{OC}$ where k is a scalar,

use a vector method to find the value of k.

$\overrightarrow{AB} = -a + b$

$\overrightarrow{BM} = 1a - \frac{1}{2}a + \frac{1}{2}b$

$= \frac{1}{2}a + \frac{1}{2}b$ ✓

$\overrightarrow{OC} = a + b$

$\frac{1}{2}a + \frac{1}{2}b = \boxed{\frac{1}{2}} \times (a + b)$

$K = \frac{1}{2}$ ✓

....

(Total for Question 20 is 3 marks)

19

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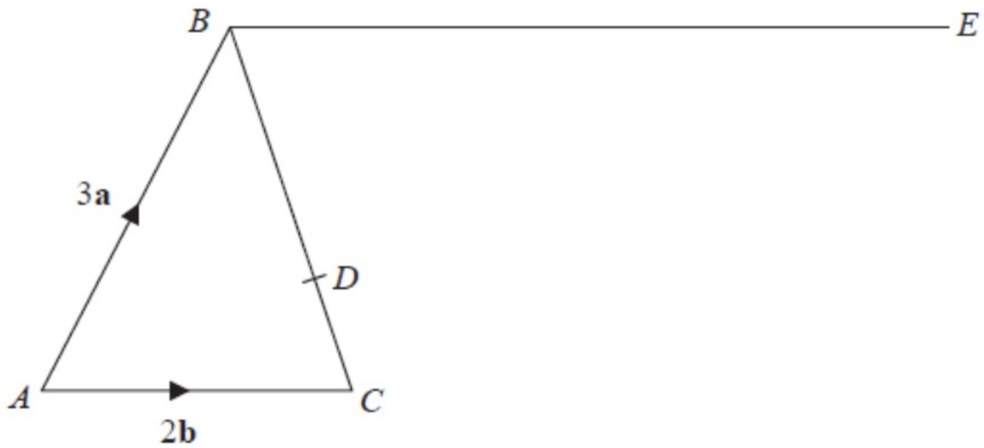

The diagram shows triangle *ABC*.

$\overrightarrow{AB} = 3\mathbf{a}$

$\overrightarrow{AC} = 2\mathbf{b}$

$\overrightarrow{BE} = 3\overrightarrow{AC}$

D is the point on *BC* such that $BD : DC = 3 : 1$

Prove that *ADE* is a straight line.

(Total for Question 19 is 4 marks)

19

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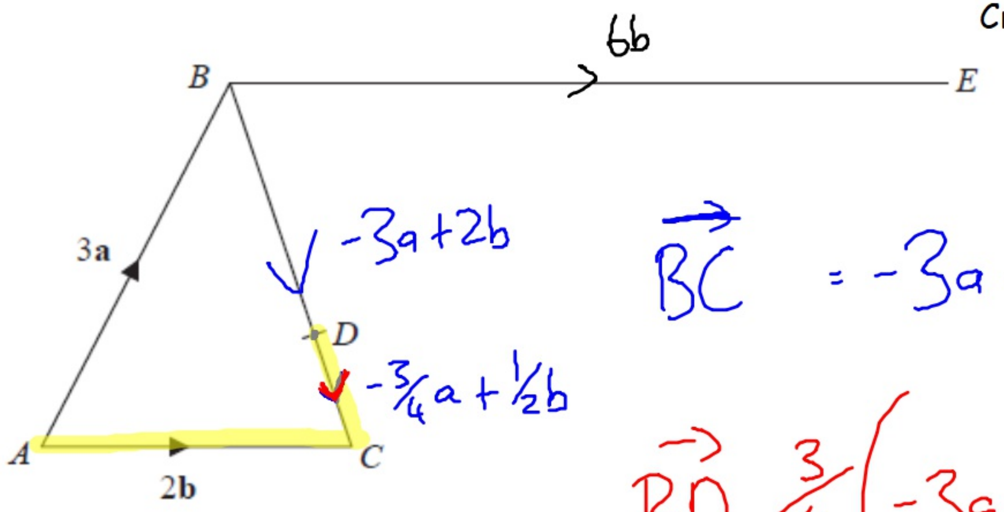

The diagram shows triangle *ABC*.

$\overrightarrow{AB} = 3\mathbf{a}$

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$\overrightarrow{BE} = 3\overrightarrow{AC}$

D is the point on *BC* such that *BD* : *DC* = 3 : 1

Prove that *ADE* is a straight line.

$\overrightarrow{BC} = -3a + 2b$

$\overrightarrow{BD} \ \frac{3}{4}(-3a+2b)$

$\overrightarrow{DC} \ \frac{1}{4}(-3a+2b) \ldots -\frac{3}{4}a+\frac{1}{2}b$

$\overrightarrow{AD} = 2b + \frac{3}{4}a - \frac{1}{2}b$

$= 1.5b + 0.75a$

$\overrightarrow{AE} = 6b + 3a$

×4

When there is a multiplier it proves that the lines are parallel.

if lines are parallel and they share a point it means that they must be on a straight line.

(Total for Question 19 is 4 marks)

19

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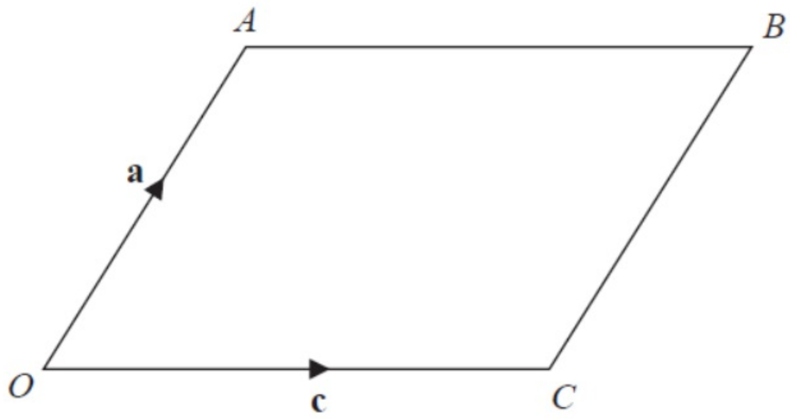

OABC is a parallelogram.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

X is the midpoint of the line *AC*.
OCD is a straight line so that $OC : CD = k : 1$

Given that $\overrightarrow{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$

find the value of *k*.

$k =$ ....

(Total for Question 19 is 4 marks)

19

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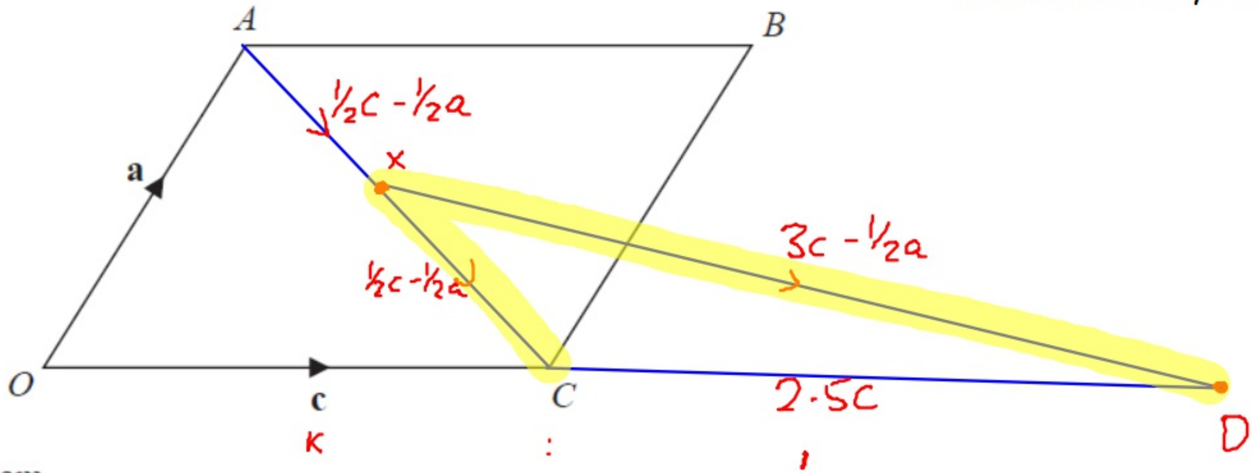
$\overrightarrow{AC} = -a + c$
$= c - a$



OABC is a parallelogram.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

X is the midpoint of the line *AC*.
OCD is a straight line so that *OC* : *CD* = *k* : 1

Given that $\overrightarrow{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$

find the value of *k*.

$\overrightarrow{CD} = -\frac{1}{2}c + \frac{1}{2}a + 3c - \frac{1}{2}a$
$= 2.5c$ ✓

$k : 1$
$1c : 2.5c$ (÷2.5)
$\frac{2}{5}c : 1c$

$1 \div 2\frac{1}{2}$
$1 \div \frac{5}{2}$
$1 \times \frac{2}{5} = \frac{2}{5}$

$k =$ $\frac{2}{5}$ ✓

(Total for Question 19 is 4 marks)

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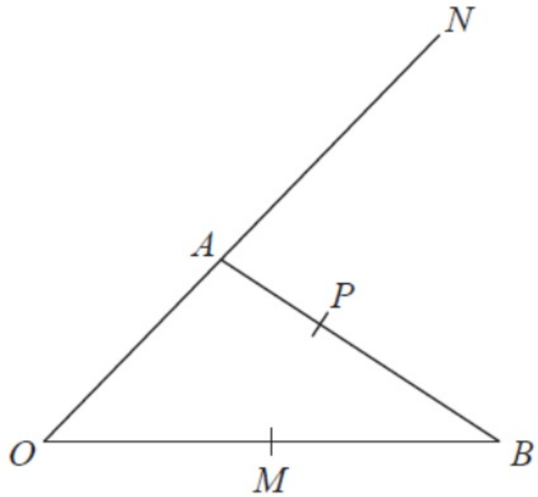
21



OAN, *OMB* and *APB* are straight lines.
$AN = 2OA$.
M is the midpoint of *OB*.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

$\overrightarrow{AP} = k\overrightarrow{AB}$ where k is a scalar quantity.

Given that *MPN* is a straight line, find the value of *k*.

....

(Total for Question 21 is 5 marks)

21

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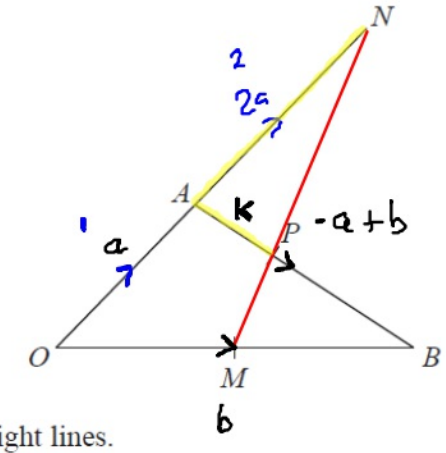

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$AN = 2OA$.
M is the midpoint of OB.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

$\overrightarrow{AP} = k\overrightarrow{AB}$ where k is a scalar quantity.

Given that MPN is a straight line, find the value of k.

$\overrightarrow{AB} = -a + b = b - a$

$\overrightarrow{MN} = -\frac{1}{2}b + 3a$

$= 3a - \frac{1}{2}b$ ✓

$\overrightarrow{PN} = -k(b - a) + 2a$

$-kb + ka + 2a$

$\overrightarrow{PN} = -kb + a(k+2)$

$\overrightarrow{MN} = 3a - \frac{1}{2}b$

$\overrightarrow{PN} = (k+2)a - kb$

$(k+2)x = 3$

$(-k)x = -\frac{1}{2}$

$(k+2)x = 3$

$x = \frac{3}{k+2}$

$-k\left(\frac{3}{k+2}\right) = -\frac{1}{2}$

$\frac{-3k}{k+2} = -\frac{1}{2}$

$\frac{3k}{k+2} = \frac{1}{2}$

$3k = \frac{1}{2}(k+2)$

$3k = \frac{1}{2}k + 1$

$3k - \frac{1}{2}k = 1$

$2\frac{1}{2}k = 1$

$k = 1 \div 2\frac{1}{2}$

$k = \frac{2}{5}$

or 0.4 ✓

OCR

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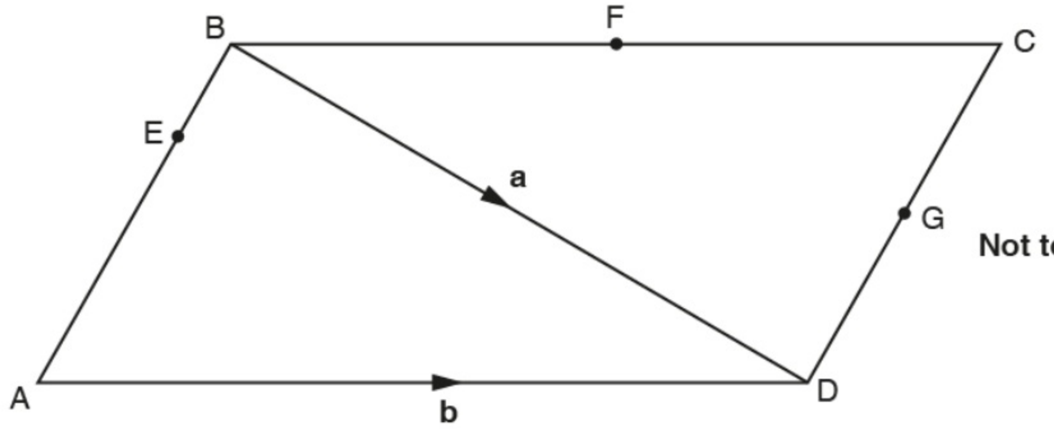

Not to scale

(b) Show that $\overrightarrow{EF} = \frac{1}{4}(3\mathbf{b} - \mathbf{a})$.

G64

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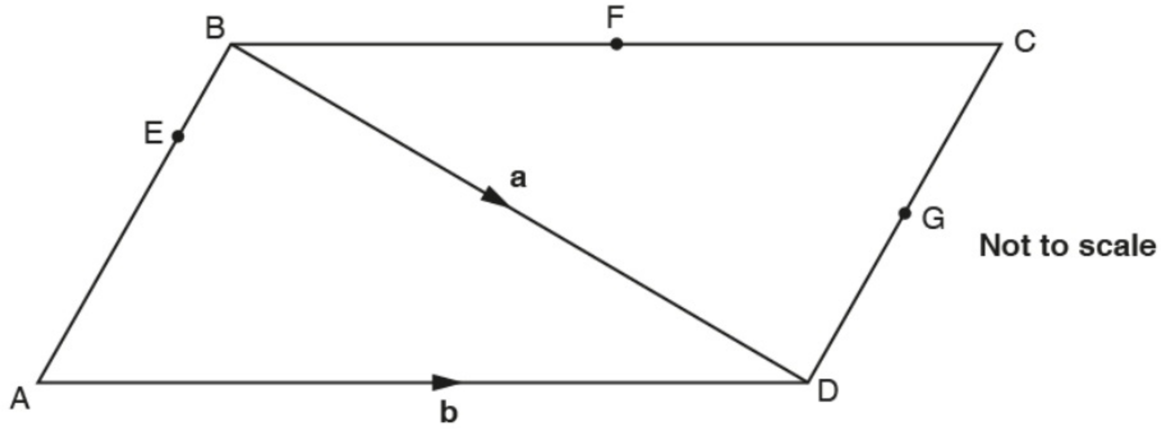

(c) Prove that $\overrightarrow{EF}$ and $\overrightarrow{AG}$ are parallel.

G64

....

....

.... **[3]**

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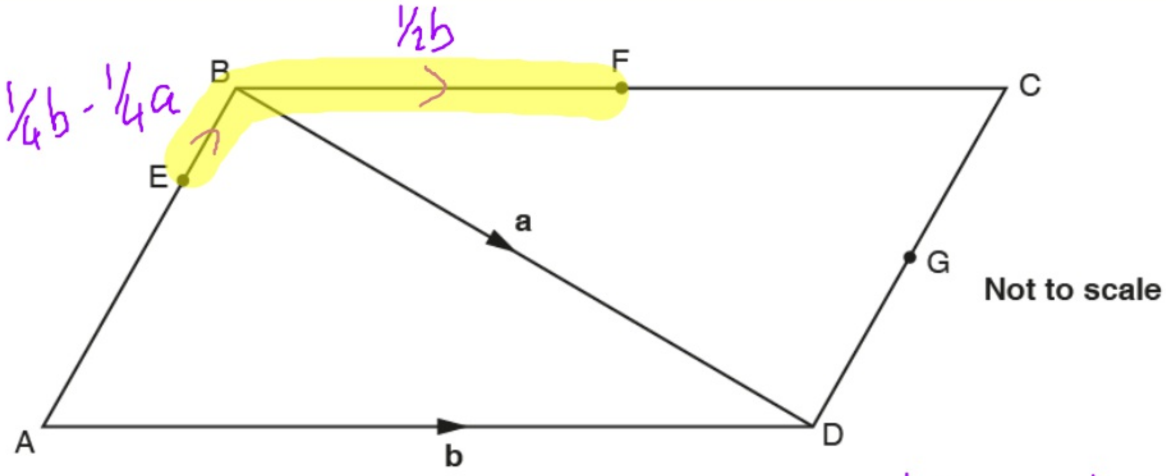

(b) Show that $\overrightarrow{EF} = \frac{1}{4}(3\mathbf{b} - \mathbf{a})$.

G64

$\frac{1}{4}b - \frac{1}{4}a + \frac{1}{2}b$

$\frac{3}{4}b - \frac{1}{4}a$

$\frac{1}{4}(3b - a)$

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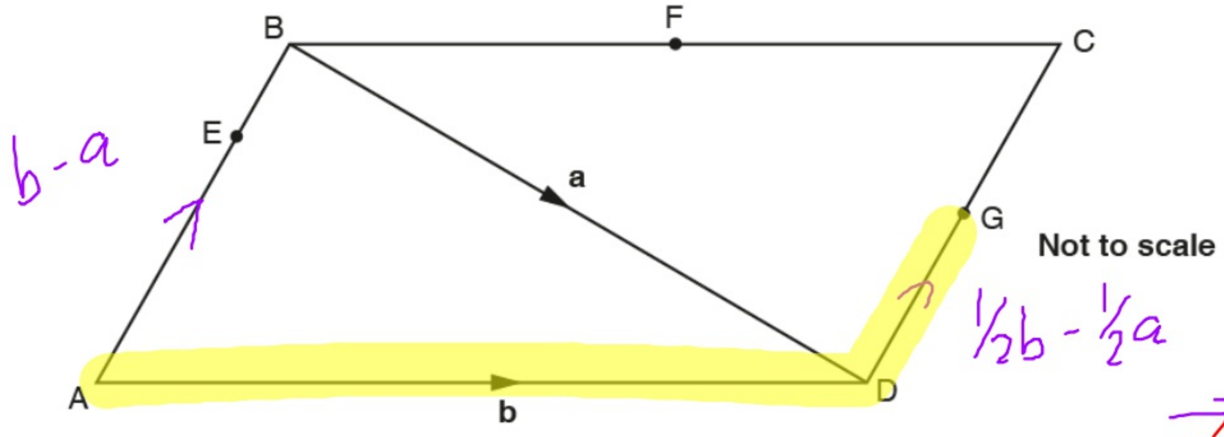

Not to scale

(c) Prove that $\overrightarrow{EF}$ and $\overrightarrow{AG}$ are parallel.

G64

$\overrightarrow{EF} = \frac{1}{4}(3b - a)$

$\overrightarrow{AG} = b + \frac{1}{2}b - \frac{1}{2}a$

$= 1\frac{1}{2}b - \frac{1}{2}a$

$= \frac{1}{2}(3b - a)$

x2

as $\overrightarrow{AG}$ is a multiple of EF (x2)

then both lines are parallel. **[3]**

AQA

23

G64

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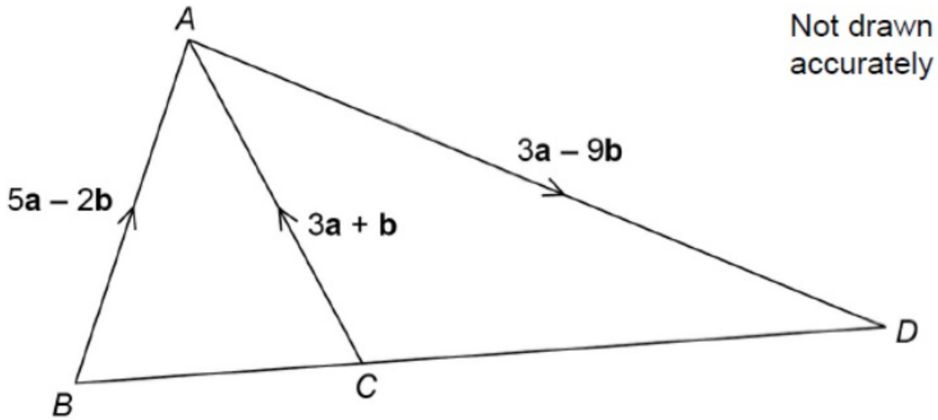

Not drawn accurately

Is *BCD* a straight line?

Show working to support your answer.

[3 marks]

Answer ______________________________

23

G64

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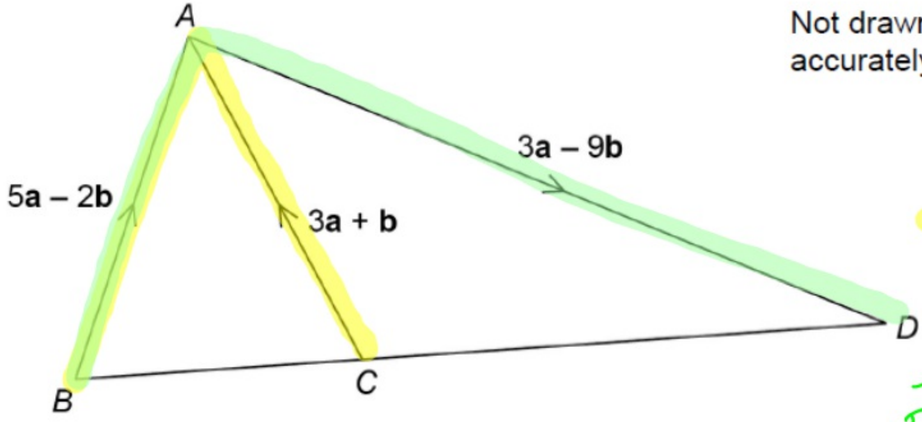

Not drawn accurately

$\overrightarrow{BC} = 5a - 2b - 3a - b$

$= 2a - 3b$

$\overrightarrow{BD} = 5a - 2b + 3a - 9b$

$8a - 11b$

Is *BCD* a straight line?

Show working to support your answer.

BC

BD

CD

$\overrightarrow{BC} = 2a - 3b$

×4 · not ×by 4

$\overrightarrow{BD} = 8a - 11b$

No, there is no multiplier so line is

Answer not straight