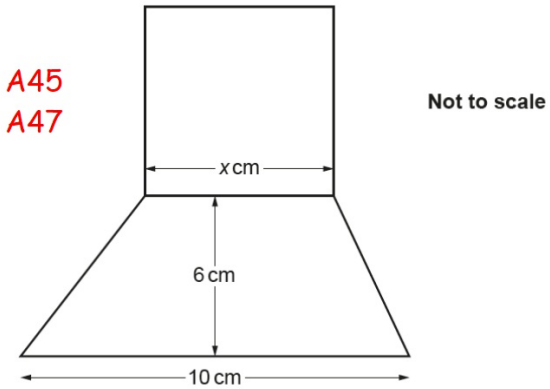


A47 (H) Quadratic Formula - Problem Solve

OCR

12 In the diagram, the square and the trapezium share a common side of length x cm.

Video created by W Neill



The area of the square is equal to the area of the trapezium.

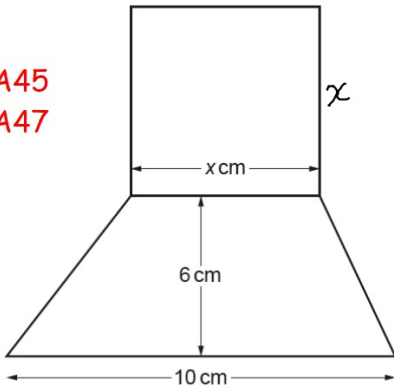
Work out the value of x .

$x =$ [6]

12 In the diagram, the square and the trapezium share a common side of length x cm.

Video created by W Neill

A45
A47



Not to scale

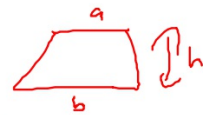
$$a = 1$$

$$b = -3$$

$$c = -30$$

Square... $x \times x = x^2$

Trapezium... $\frac{1}{2}(a+b)h$



$$\frac{1}{2}(x+10)6$$

$$3(x+10) \dots 3x+30$$

$$x^2 = 3x + 30$$

$$4x + x - 30$$

$$x^2 - 3x - 30 = 0$$

$$= -120$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The area of the square is equal to the area of the trapezium.

Work out the value of x .

~~$$\frac{3 - \sqrt{129}}{2}$$~~

neg

$$\frac{3 + \sqrt{129}}{2}$$

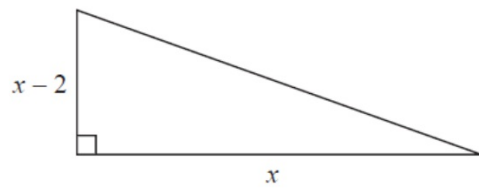
$$x = \frac{3 \pm \sqrt{9 - (-120)}}{2} = \frac{3 \pm \sqrt{129}}{2}$$

$$x = \dots 7.18 \text{ cm} \checkmark [6]$$

Edexcel

19 Here is a right-angled triangle.

Video created by W Neill



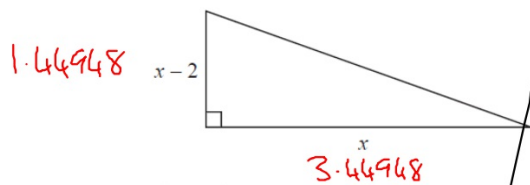
All measurements are in centimetres.
The area of the triangle is 2.5 cm^2 .

Find the perimeter of the triangle.
Give your answer correct to 3 significant figures.
You must show all of your working.

..... cm

(Total for Question 19 is 6 marks)

19 Here is a right-angled triangle.



All measurements are in centimetres.
The area of the triangle is 2.5 cm².

Find the perimeter of the triangle.
Give your answer correct to 3 significant figures.
You must show all of your working.

Area

$$\frac{B \times H}{2} = 2.5$$

$$\frac{x(x-2)}{2}$$

$$\dots \frac{x^2 - 2x}{2} = 2.5$$

$$\dots \frac{1}{2}x^2 - x = 2.5$$

$$\frac{x^2 - 2x}{2} = 2.5$$

$$x^2 - 2x = 5$$

$$x^2 - 2x - 5 = 0$$

$$a=1$$

$$b=-2$$

$$c=-5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{2 \pm \sqrt{4 - 20}}{2}$$

$$\frac{2 \pm \sqrt{24}}{2}$$

$$4 \times 1 \times -5$$

$$x = 3.44948$$

$$-1.44$$

Perimeter

Pythagoras

$$2.10099$$

$$13.9999$$

$$1.44948$$

$$3.7416$$

$$3.44948$$

$$11.898912$$

Perimeter

$$8.6406$$

$$8.64 \checkmark \text{ cm}$$

(Total for Question 19 is 6 marks)

21 Given that

Video created by W Neill

$$2x - 1 : x - 4 = 16x + 1 : 2x - 1$$

find the possible values of x .

.....
(Total for Question 21 is 5 marks)

21 Given that

$$2x-1 : x-4 = 16x+1 : 2x-1$$

find the possible values of x .

$$\frac{2x-1}{x-4} = \frac{16x+1}{2x-1}$$

$$(2x-1)(2x-1) = (16x+1)(x-4)$$

$$4x^2 - 2x - 2x + 1 = 16x^2 - 64x + 1x - 4$$

$$4x^2 - 4x + 1 = 16x^2 - 63x - 4$$

$$0 = 16x^2 - 4x^2 - 63x + 4x - 4 - 1$$

$$0 = 12x^2 - 59x - 5$$

↗
Quadratic

$$4ac = 4 \times 12 \times -5 = -240$$

$$3:4 = 6:8$$

$$\frac{3}{4} = \frac{6}{8} \checkmark$$

$$12x^2 - 59x - 5 = 0$$

$$a = 12 \\ b = -59 \\ c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{59 \pm \sqrt{3481 - 240}}{24}$$

$$x = \frac{59 \pm \sqrt{3721}}{24}$$

$$x = \frac{59 \pm 61}{24}$$

$$5 \quad \swarrow \quad \searrow \quad -\frac{1}{12}$$

(Total for Question 21 is 5 marks)

22 There are y black socks and 5 white socks in a drawer.

Video created by W Neill

Joshua takes at random two socks from the drawer.

The probability that Joshua takes one white sock and one black sock is $\frac{6}{11}$

(a) Show that $3y^2 - 28y + 60 = 0$

(4)

(b) Find the probability that Joshua takes two black socks.

.....
(3)

(Total for Question 22 is 7 marks)

22 There are y black socks and 5 white socks in a drawer.

Joshua takes at random two socks from the drawer.

The probability that Joshua takes one white sock and one black sock is $\frac{6}{11}$

(a) Show that $3y^2 - 28y + 60 = 0$

$$\frac{10y}{y^2 + 9y + 20} = \frac{6}{11}$$

$$\begin{aligned} 110y &= 6(y^2 + 9y + 20) \\ 110y &= 6y^2 + 54y + 120 \\ 0 &= 6y^2 + 54y - 110y + 120 \end{aligned}$$

$$\begin{aligned} 6y^2 - 54y + 120 &= 0 \\ 3y^2 - 28y + 60 &= 0 \end{aligned}$$

and = X
or = f

y = black
S = white
Total = y + 5

(black and white) or (white and black)

$$\left(\frac{y}{y+5} \times \frac{5}{y+4}\right) + \left(\frac{5}{y+5} \times \frac{y}{y+4}\right)$$

$$\frac{5y}{y^2 + 9y + 20} + \frac{5y}{y^2 + 9y + 20} = \frac{10y}{y^2 + 9y + 20}$$

(b) Find the probability that Joshua takes two black socks.

$$\begin{aligned} 3y^2 - 28y + 60 &= 0 \\ (3y - 10)(y - 6) &= 0 \\ \begin{array}{r} +60 \\ -10 \quad -6 \end{array} & \\ 3y - 10 = 0 & \quad y = 6 \\ 3y = 10 & \\ y = 10/3 & \dots \text{don't use} \end{aligned}$$

$$\begin{aligned} \rightarrow \text{Black}(y) &= 6 \\ B \text{ and } B & \\ \frac{6}{11} \times \frac{5}{11} &= \frac{30}{110} = \frac{3}{11} \quad \checkmark \end{aligned}$$

(Total for Question 22 is 7 marks)

$$\begin{aligned} (y+5)(y+4) & \\ y^2 + 5y + 4y + 20 & \\ y^2 + 9y + 20 & \end{aligned}$$

Video created by W Neill

21 The length of a rectangle is the same as the length of each side of a square.

The length of the rectangle is 4 cm more than 3 times the width of the rectangle.

The area of the square is 66 cm^2 more than the area of the rectangle.

Find the length and the width of the rectangle.

You must show all your working.

(Total for Question 21 is 6 marks)

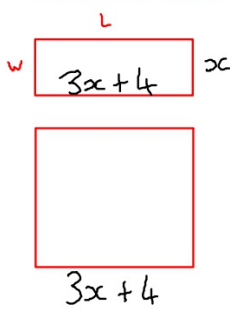
21 The length of a rectangle is the same as the length of each side of a square.

The length of the rectangle is 4 cm more than 3 times the width of the rectangle.

The area of the square is 66 cm^2 more than the area of the rectangle.

Find the length and the width of the rectangle.

You must show all your working.



$$\begin{aligned} \text{Rectangle (Area)} \\ x(3x+4) \\ = 3x^2 + 4x \end{aligned}$$

$$\begin{aligned} \text{Square (Area)} \\ (3x+4)(3x+4) \\ = 9x^2 + 12x + 12x + 16 \\ = 9x^2 + 24x + 16 \end{aligned}$$

$$9x^2 + 24x + 16 = 3x^2 + 4x + 66$$

$$9x^2 - 3x^2 + 24x - 4x + 16 - 66 = 0$$

$$6x^2 + 20x - 50 = 0$$

$$3x^2 + 10x - 25 = 0$$

$$(3x - 5)(x + 5) = 0$$

$$\begin{array}{l} 3x - 5 = 0 \quad \quad \quad x + 5 = 0 \\ 3x = 5 \quad \quad \quad x = -5 \\ x = \frac{5}{3} \text{ or } 1\frac{2}{3} \end{array}$$

$$\begin{aligned} 3 \times \frac{5}{3} &= \frac{15}{3} = 5 \\ 3 \times 1\frac{2}{3} &= 5 \\ 5 + 4 &= 9 \end{aligned}$$

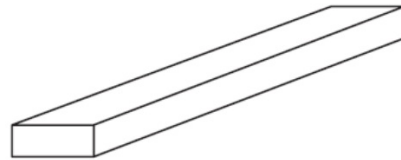
Ans
Width = $1\frac{2}{3} \text{ cm}$
Length = 9 cm ✓

(Total for Question 21 is 6 marks)

22 A solid cuboid has a volume of 40 cm^3
The cuboid has a total surface area of 100 cm^2
One edge of the cuboid has length 2 cm .

Find the length of a diagonal of the cuboid.
Give your answer correct to 3 significant figures.

Created by W Neill

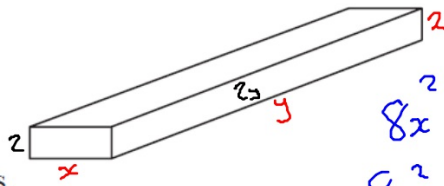


.....cm

(Total for Question 22 is 6 marks)

22 A solid cuboid has a volume of 40 cm^3
 The cuboid has a total surface area of 100 cm^2
 One edge of the cuboid has length 2 cm .

Find the length of a diagonal of the cuboid.
 Give your answer correct to 3 significant figures.



Volume: $2xy = 40 \text{ cm}^3$

SA: $2x + 2x + 2y + 2y + xy + xy = 100 \text{ cm}^2$
 $4x + 4y + 2xy = 100 \text{ cm}^2$

$2xy = 40$
 $4x + 4y + 2xy = 100$

$4x + 4y = 60 \dots 2xy = 40$
 $4x + 4\left(\frac{40}{2x}\right) = 60 \quad y = \frac{40}{2x}$

$4x + \frac{160}{2x} = 60$

$8x^2 + 160 = 120x$

(-8)

$a = 1$
 $b = -15$
 $c = +20$

$8x^2 + 160 = 120x$
 $8x^2 - 120x + 160 = 0$
 $x^2 - 15x + 20 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $\frac{15 \pm \sqrt{225 - 80}}{2}$
 $\frac{15 \pm \sqrt{145}}{2}$

$x = 1.479 \text{ or } 13.520$

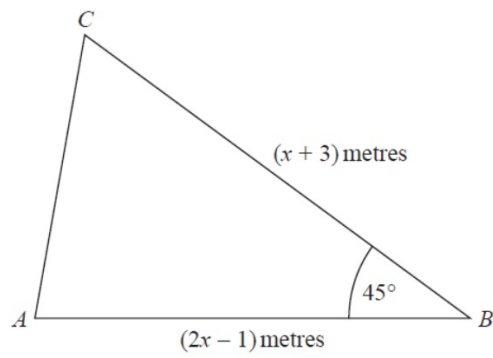
$2xy = 40$

$y = \frac{40}{2(1.479)} \dots \text{cm}$

(Total for Question 22 is 6 marks)

15

Video created by W Neill

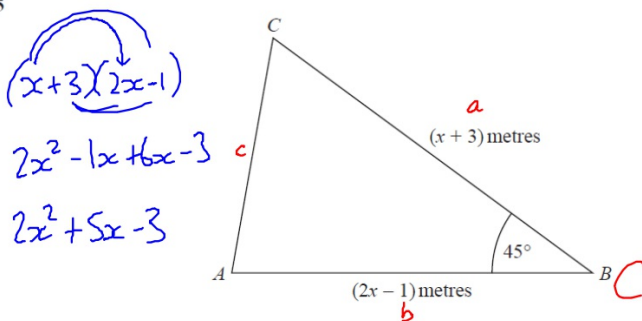


The area of triangle ABC is $6\sqrt{2}$ m².

Calculate the value of x .

Give your answer correct to 3 significant figures.

.....
(Total for Question 15 is 5 marks)



The area of triangle ABC is $6\sqrt{2} \text{ m}^2$.

Calculate the value of x .

Give your answer correct to 3 significant figures.

$$a=2$$

$$b=5$$

$$C=27$$

$$4 \times 2x - 27$$

$$-216$$

$$2x^2 + 5x - 27 = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{25 - (-216)}}{4}$$

$$x = \frac{-5 \pm \sqrt{241}}{4}$$

$$\frac{-5 + \sqrt{241}}{4}$$

$$2.63 \checkmark$$

$$\frac{1}{2}ab \sin C$$

$$\frac{1}{2}(x+3)(2x-1) \sin 45^\circ$$

$$\frac{1}{2}(2x^2 + 5x - 3) \frac{\sqrt{2}}{2} = 6\sqrt{2}$$

$$(2x^2 + 5x - 3) \frac{\sqrt{2}}{2} = 12\sqrt{2}$$

$$2x^2 + 5x - 3 = \frac{24\sqrt{2}}{\sqrt{2}}$$

$$2x^2 + 5x - 3 = 24$$

$$2x^2 + 5x - 3 - 24 = 0$$

$$2x^2 + 5x - 27 = 0$$

(Total for Question 15 is 5 marks)

Created by W Neill

19 Prove algebraically that the straight line with equation $x - 2y = 10$ is a tangent to the circle with equation $x^2 + y^2 = 20$

(Total for Question 19 is 5 marks)

19 Prove algebraically that the straight line with equation $x - 2y = 10$ is a tangent to the circle with equation $x^2 + y^2 = 20$

$$x - 2y = 10$$

$$x = 2y + 10$$

$$(2y + 10)^2$$

$$(2y + 10)(2y + 10)$$

$$4y^2 + 20y + 20y + 100$$

$$4y^2 + 40y + 100$$

$$4y^2 + 40y + 100 + y^2 - 20 = 0$$

$$5y^2 + 40y + 80 = 0$$

$$(5y + 20)(y + 4) = 0$$

$$5y + 20 = 0 \quad \left. \begin{array}{l} y + 4 = 0 \\ y = -4 \end{array} \right\}$$

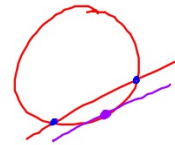
$$5y = -20$$

$$y = -4$$

y coordinates are equal for the point of intersection, meaning there is only 1 point of intersection

This means it must be a tangent as a tangent meets at 1 point.

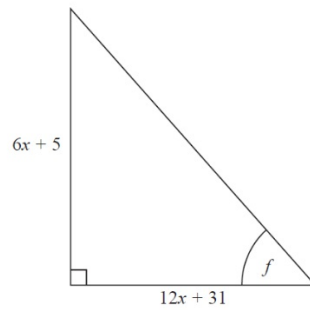
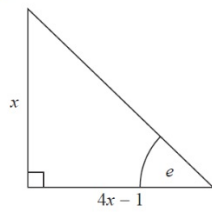
(Total for Question 19 is 5 marks)



19 Here are two right-angled triangles.

Video created by W Neill

A47
A54



Given that

$$\tan e = \tan f$$

find the value of x .

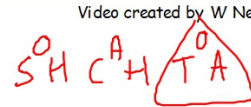
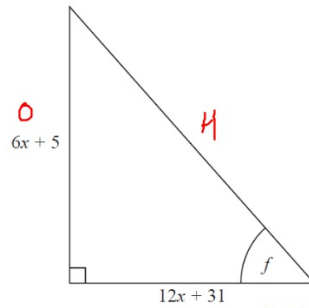
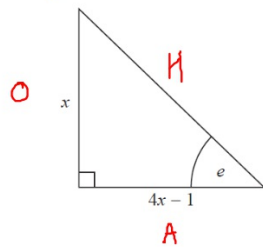
You must show all your working.

.....
(Total for Question 19 is 5 marks)

19 Here are two right-angled triangles.

Video created by W Neill

A47
A54



Given that

$$\tan e = \tan f$$

find the value of x .

You must show all your working.

$$\frac{x}{4x-1} = \frac{6x+5}{12x+31}$$

$$x(12x+31) = (6x+5)(4x-1)$$

$$\Rightarrow 12x^2 + 31x = 24x^2 - 6x + 20x - 5$$

$$12x^2 + 31x = 24x^2 + 14x - 5$$

$$0 = 24x^2 + 14x - 31x - 12x^2 - 5$$

$$0 = 12x^2 - 17x - 5$$

~~$$\tan \frac{x}{4x-1} = \tan \frac{6x+5}{12x+31}$$~~

$$12x^2 - 17x - 5 = 0$$

$$a = 12$$

$$b = -17$$

$$c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{17 \pm \sqrt{289 - (-240)}}{24}$$

$$x = \frac{17 \pm 23}{24}$$

$$\xrightarrow{\text{Ans}} \frac{17+23}{24}$$

$$x = \frac{5}{3} \checkmark$$

(Total for Question 19 is 5 marks)

AQA

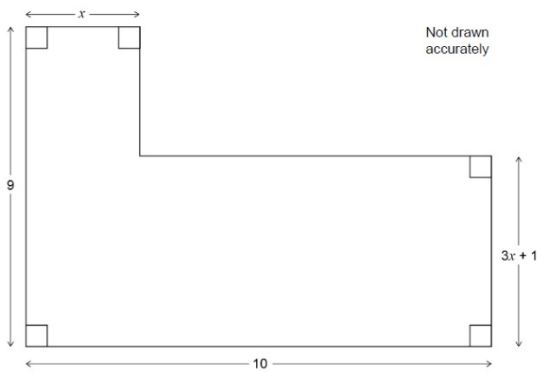
26

Here is an L-shape.

Video created by W Neill

A47

All dimensions are in centimetres.



The area of the L-shape is 65 cm^2

Work out the value of x . [6 marks]

$$-3x^2 + 38x + 10 = 65$$

$$3x^2 + 38x + 10 - 65 = 0$$

$$3x^2 + 38x - 55 = 0$$

5
38
-55

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-38 \pm \sqrt{1444 - 660}}{-6}$$

$\frac{5}{3}$

$$x = \frac{-38 \pm \sqrt{784}}{-6}$$

Answer

$\frac{5}{3}, \text{ (11)} \times -6$

26

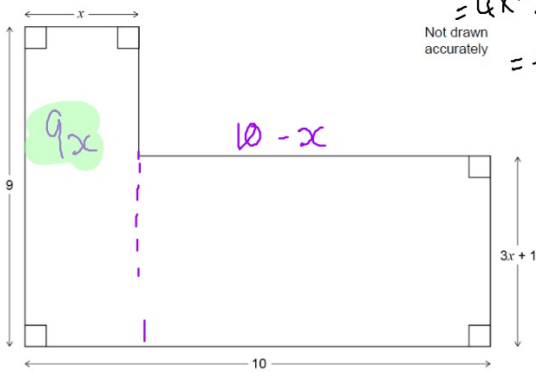
A47

Here is an L-shape.

All dimensions are in centimetres.

$$b^2 - 4ac = 4x^2 - 3x - 55 = +$$

Not drawn accurately



$$(10-x)(3x+1)$$

$$30x + 10 - 3x^2 - 1x$$

$$-3x^2 + 29x + 10 + 9x$$

$$-3x^2 + 38x + 10$$

Video created by W Neill

The area of the L-shape is 65 cm^2

Work out the value of x . [6 marks]

$$-3x^2 + 38x + 10 = 65$$

$$-3x^2 + 38x + 10 - 65 = 0$$

$$-3x^2 + 38x - 55 = 0$$

$$a = -3$$

$$b = 38$$

$$c = -55$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-38 \pm \sqrt{1444 - 660}}{-6}$$

$$x = \frac{-38 \pm \sqrt{784}}{-6}$$

$$x = \frac{5}{3}$$

Answer

$$\frac{5}{3}, \text{ (11) } -6$$